Lund University

CHALMERS UNIVERSITY OF TECHNOLOGY

WASP WALLENBERG AI, AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM

Certifiably Optimal Anisotropic Rotation Averaging

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[1] Lund University [2] Chalmers University of Technology

ICCV OCT 19-23, 2025 HONOLULU HAWAII

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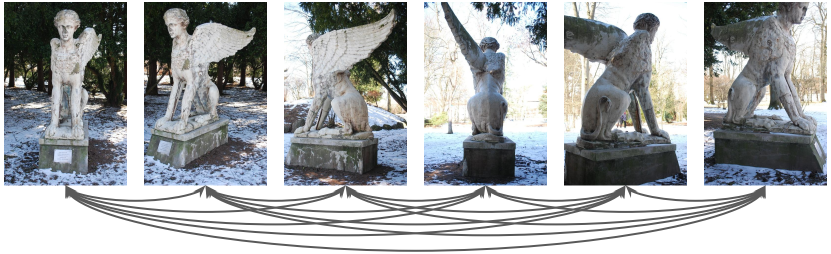

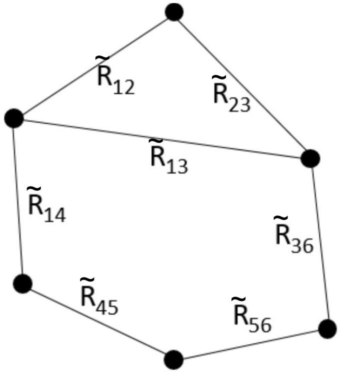

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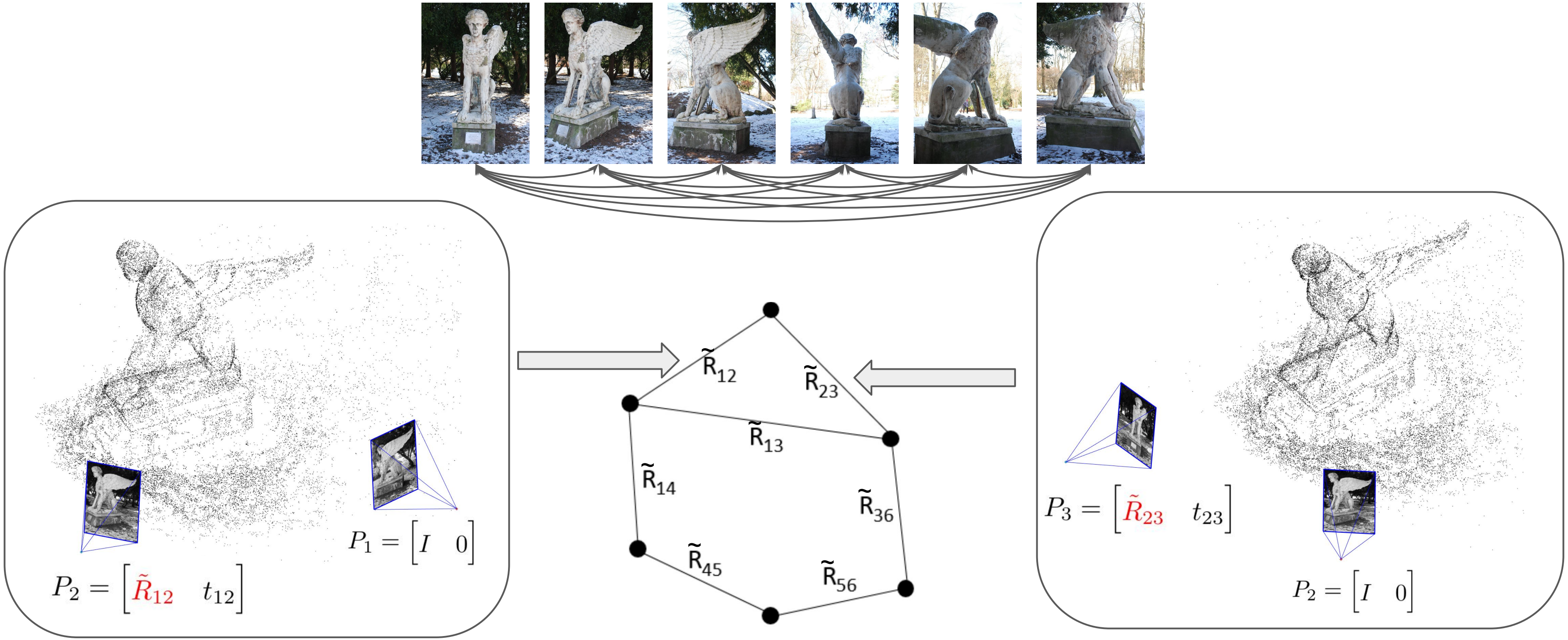

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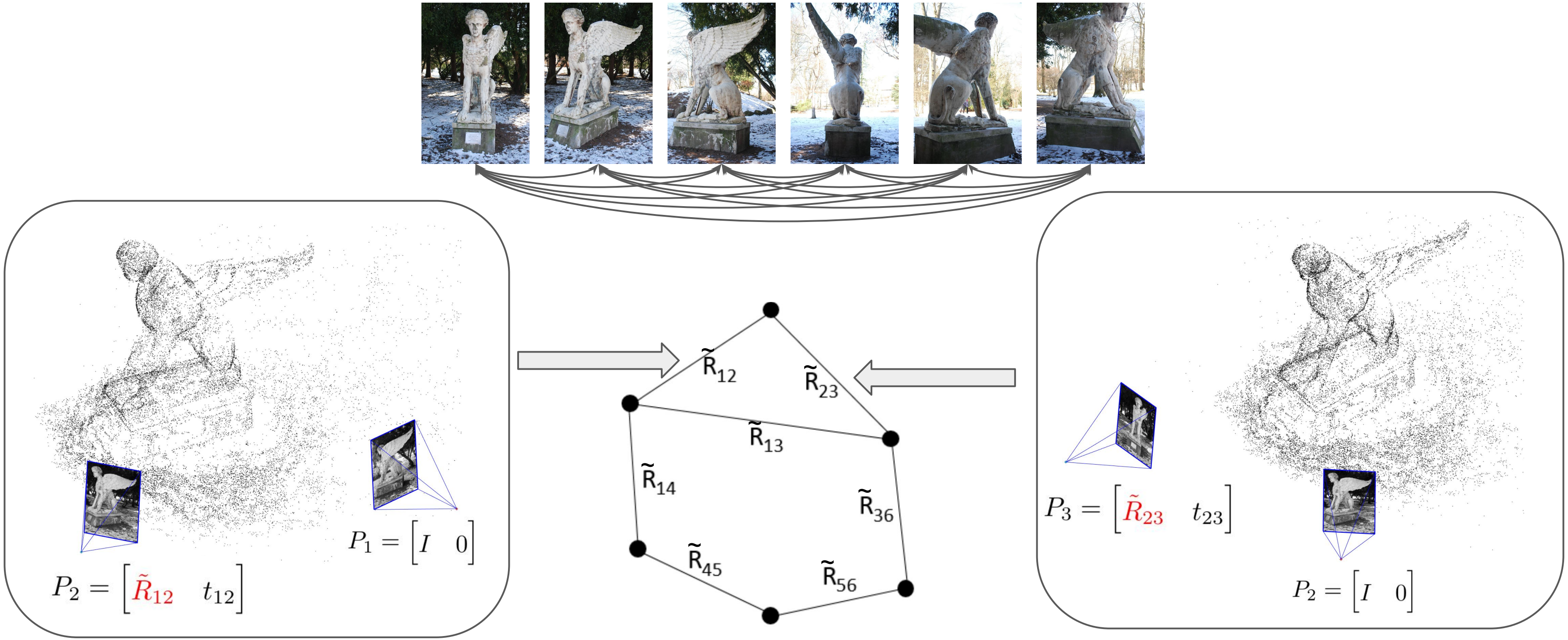

Given relative rotations (edges).
Relative pose solutions (local coord. sys.)

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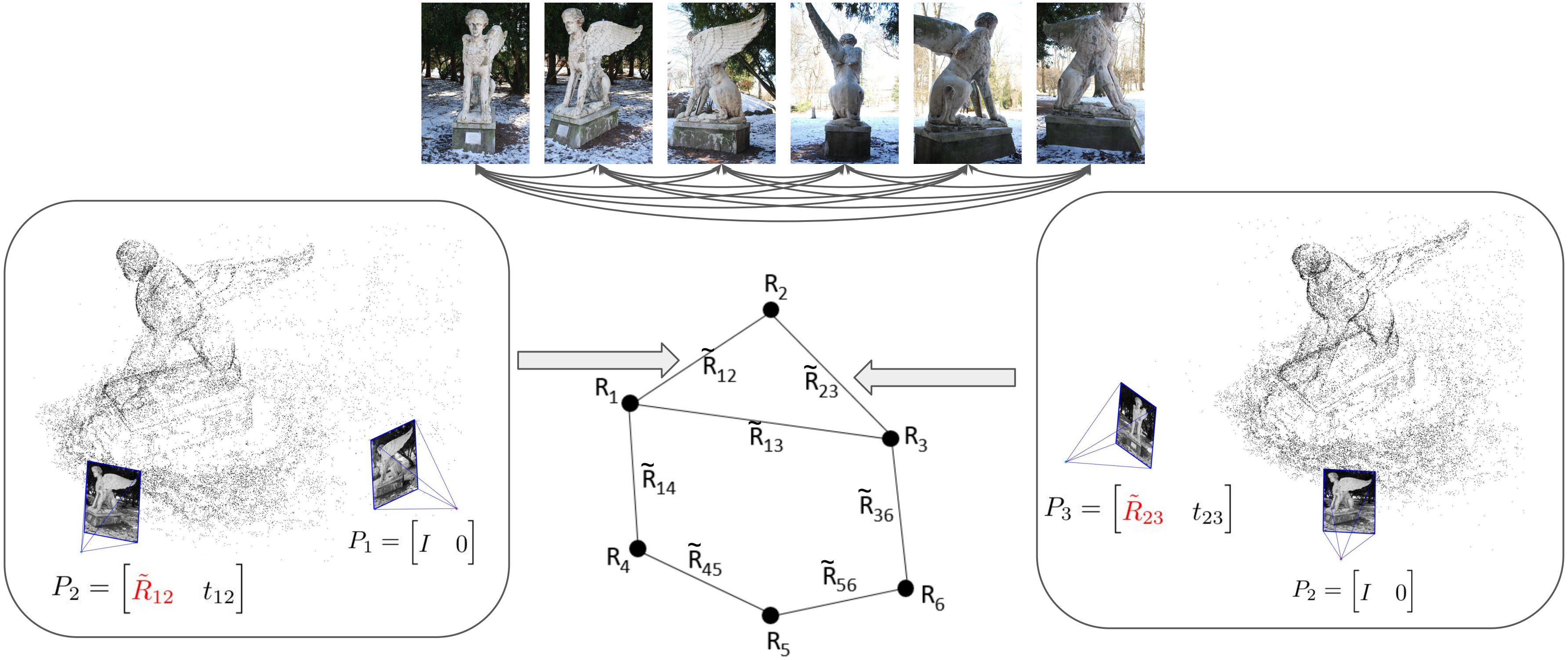
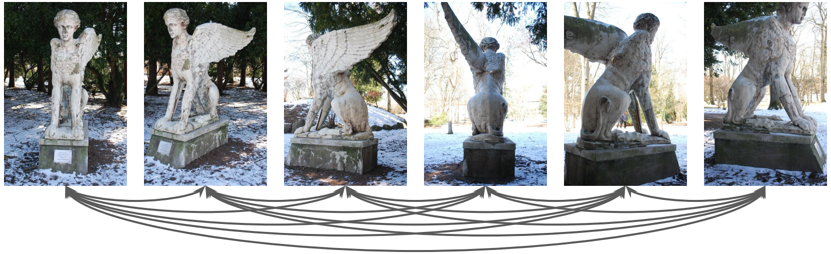


Given relative rotations (edges).
Relative pose solutions (local coord. sys.)

Compute camera orientations (nodes).
Rotations in global coord. sys.

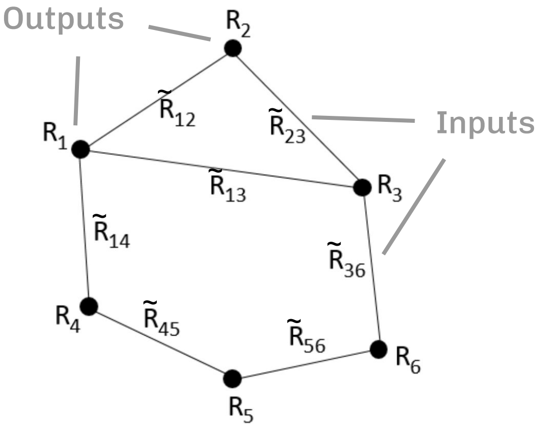
Problem formulation

Given $\{\tilde{R}_{ij}\}$, estimate $\{R_i\}$



$$\min_{\{R_i\}} \sum_{(i,j)\in E} d^2(\tilde{R}_{ij} R_j, R_i)$$

$$R_i \in SO(3) = \{R \in \mathbb{R}^{3\times 3}; R^T R = I, \det(R) = 1\}$$



Isotropic RA: chordal distances

$$d^2(\tilde{R}_{ij}R_j, R_i) = \|\tilde{R}_{ij}R_j - R_i\|_F^2 = \underbrace{\|\tilde{R}_{ij}R_i\|_F^2}_{=3} + \underbrace{\|R_j\|_F^2}_{=3} - 2\underbrace{\operatorname{tr}(\tilde{R}_{ij}R_jR_i^\top)}_{=1+2\cos(\phi)}$$

Residual rotation angle. Invariant to residual rotation axis.

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$$d^2(\tilde{R}_{ij}R_j, R_i) = \|\tilde{R}_{ij}R_j - R_i\|_F^2 = \underbrace{\|\tilde{R}_{ij}R_i\|_F^2}_{=3} + \underbrace{\|R_j\|_F^2}_{=3} - 2\underbrace{\mathrm{tr}(\tilde{R}_{ij}R_jR_i^\top)}_{=1+2\cos(\phi)}$$

Residual rotation angle. Invariant to residual rotation axis.

In matrix form:

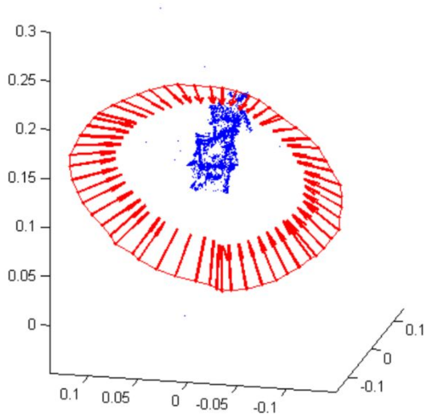
$$\min_{\{R_i \in SO(3)\}} -\mathrm{tr}(\tilde{\mathbf{R}}\mathbf{R}\mathbf{R}^\top)$$

$$\mathbf{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_n \end{pmatrix} \qquad \tilde{\mathbf{R}} = \begin{pmatrix} 0 & \tilde{R}_{12}^\top & \tilde{R}_{13}^\top & \dots & \tilde{R}_{1n}^\top \\ \tilde{R}_{12} & 0 & \tilde{R}_{23}^\top & \dots & \tilde{R}_{2n}^\top \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{1n} & \tilde{R}_{2n} & \tilde{R}_{3n} & \dots & 0 \end{pmatrix}$$

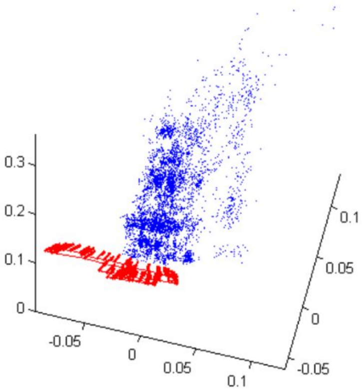
Isotropic RA: non-convex problem

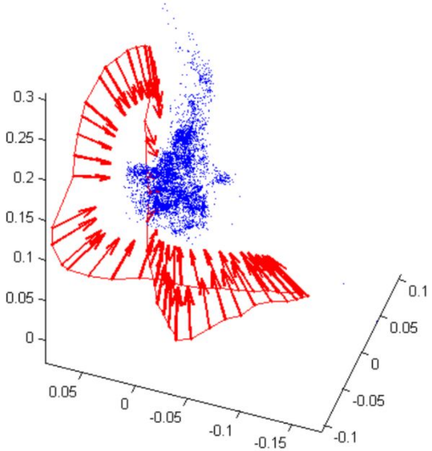
$$\min_{\{R_i \in SO(3)\}} -\operatorname{tr}(\tilde{\mathbf{R}}\mathbf{R}\mathbf{R}^\top)$$

Global minimum



Local minima:





[Eriksson et al. Rotation Averaging and Strong Duality. CVPR 2018]

Isotropic RA: convex relaxation

Drop the determinant constraint:

$$\min_{\{R_i \in O(3)\}} \; -\operatorname{tr}(\tilde{\mathbf{R}}\mathbf{R}\mathbf{R}^\top)$$

Take Lagrange-dual twice ⟶ Linear SDP:

SDP-O(3)-ISO:

$$\min_{\mathbf{X} \succeq 0} \; -\operatorname{tr}(\tilde{\mathbf{R}}\mathbf{X})$$

$$\text{s.t. } \mathbf{X}_{ii} = \mathrm{I}_3$$

If $\operatorname{rank}(\mathbf{X}) = \mathbf{3}$, extract $\mathbf{R}$ from $\mathbf{X} = \mathbf{R}\mathbf{R}^\top$.
Certifiably optimal solution

[Eriksson et al. Rotation Averaging and Strong Duality. CVPR 2018]

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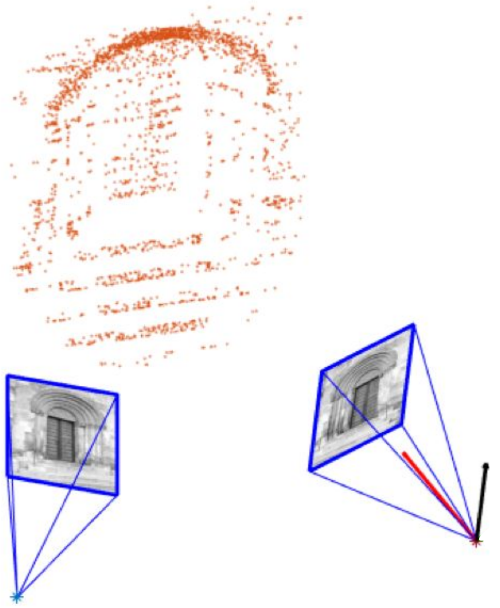
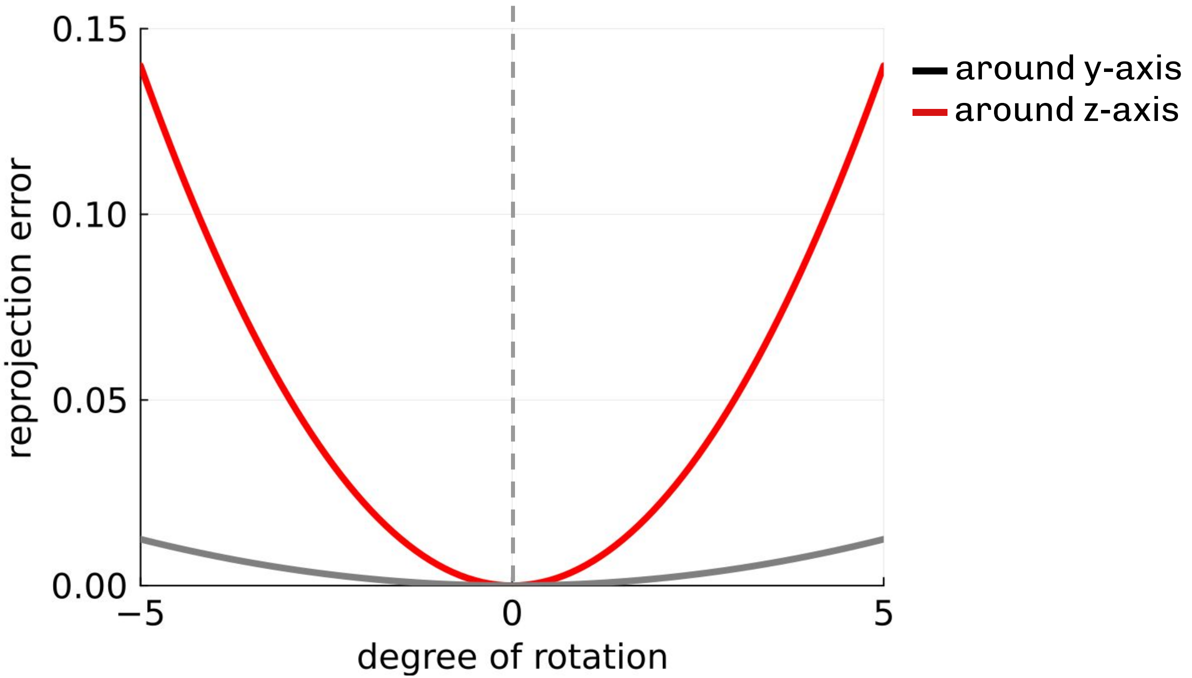
Uncertainty of two-view optimized rotations



Lateral motion



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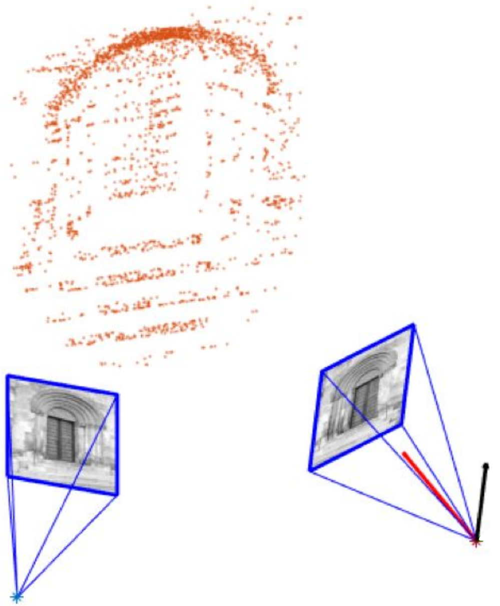

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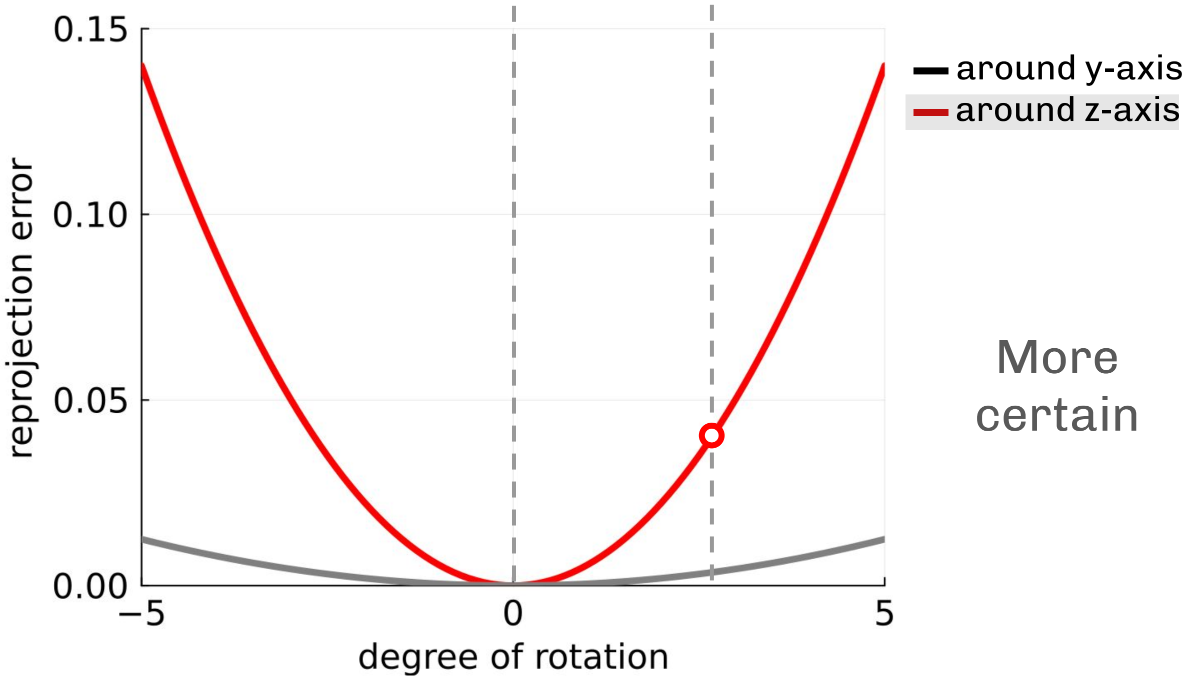
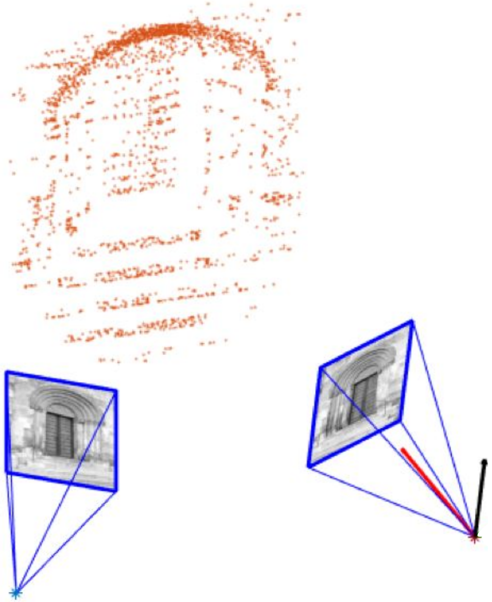
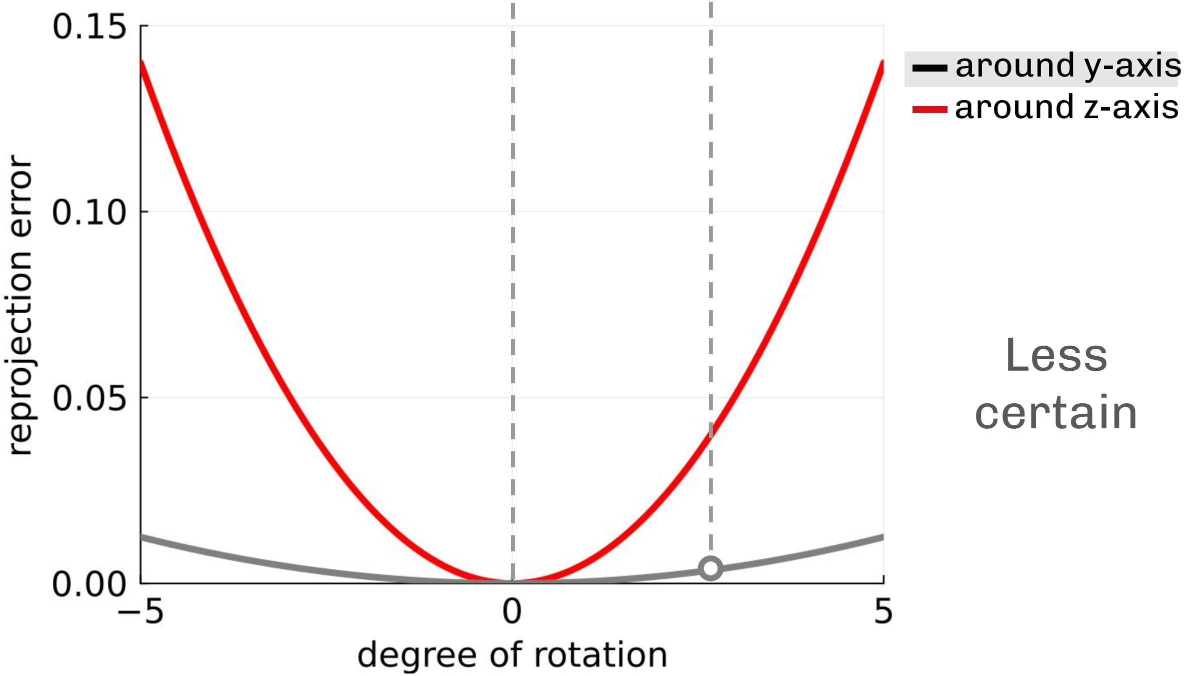


More certain

Uncertainty of two-view optimized rotations

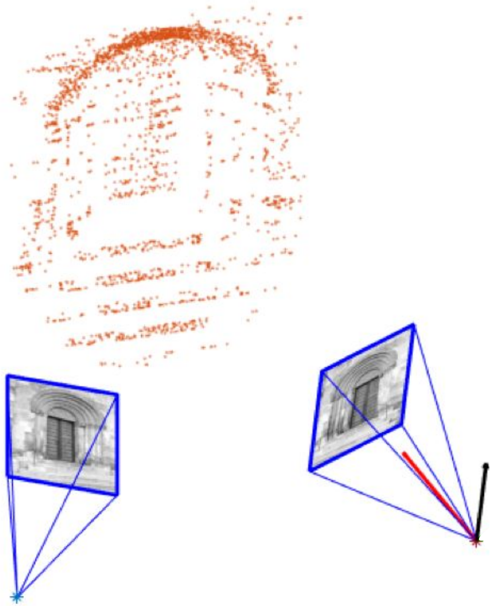


Lateral motion

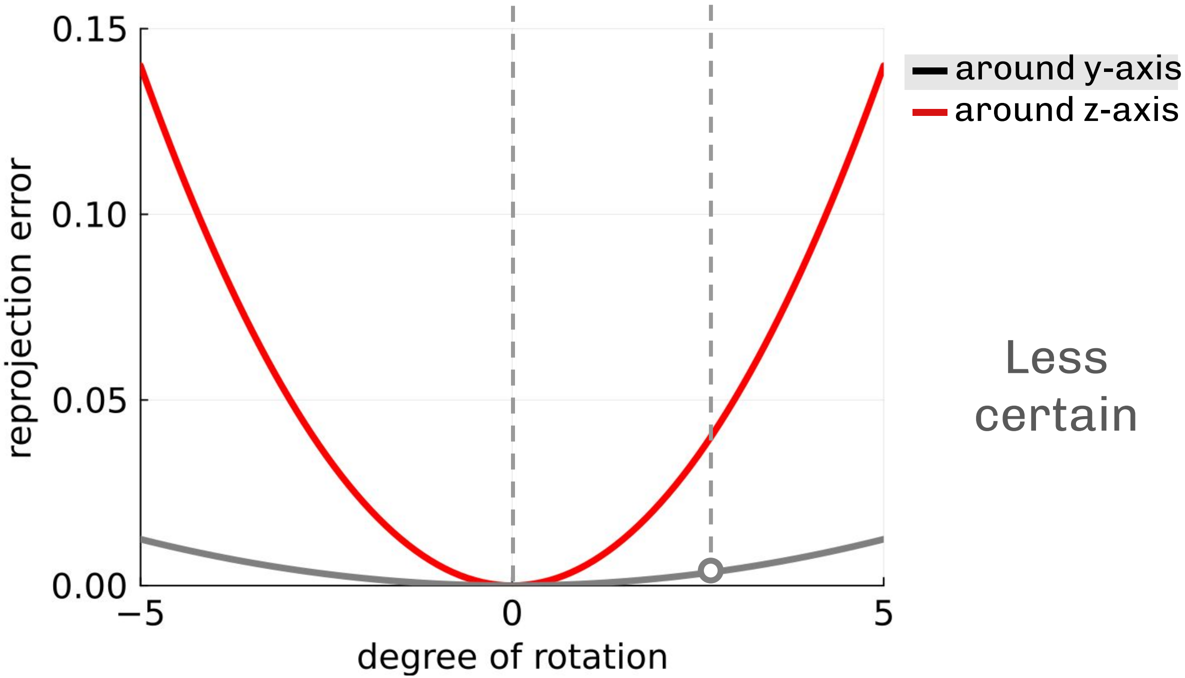


Less certain

Uncertainty of two-view optimized rotations



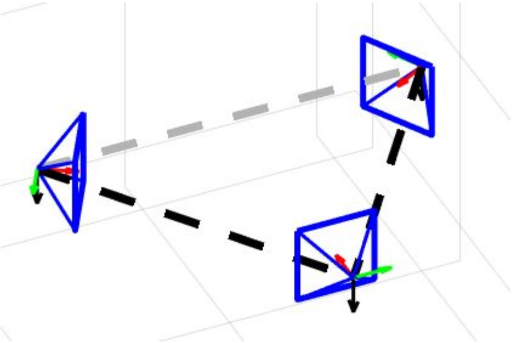
Lateral motion



Less certain

Propagate to averaging stage. Favour deviations in the directions of high uncertainty!

Toy example and sneak peek



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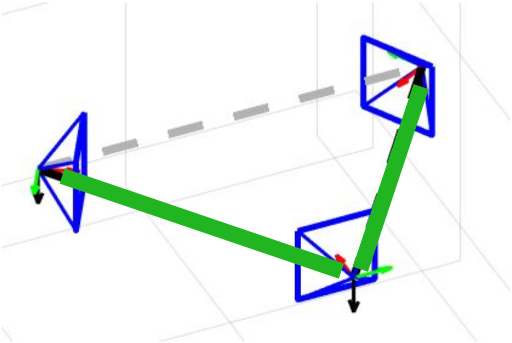

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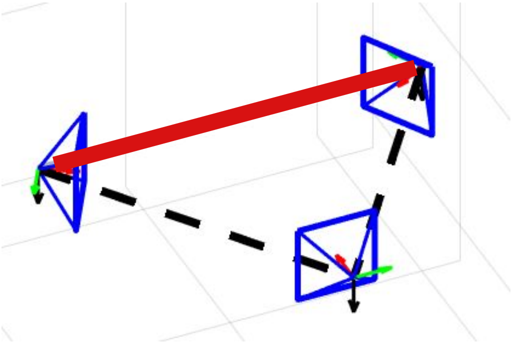

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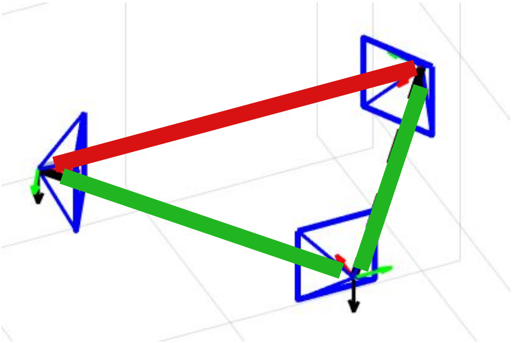
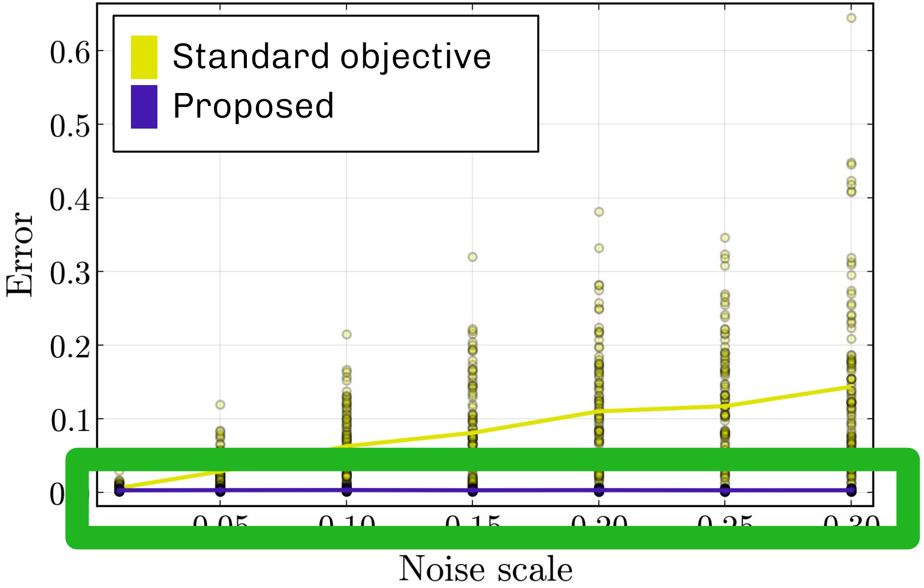


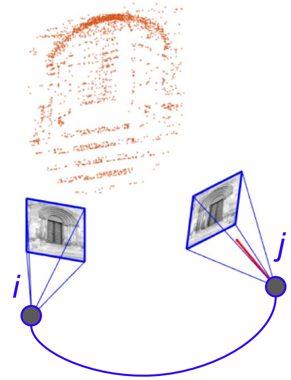


Result #1: uncertainty propagation for RA

Around local min. $\tilde{R}$, the objective *is* approximated as

$$\Delta\omega^\top H \Delta\omega$$

where $Q = e^{[\Delta\omega]_\times} \tilde{R},\quad \Delta\omega$ — angle axis vector.



(i,j dropped)

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$$\Delta\boldsymbol{\omega}^\top H \Delta\boldsymbol{\omega} \overset{?}{\approx} -\operatorname{tr}\left(M\tilde{R}Q^\top\right) \quad \text{(+const.)}$$

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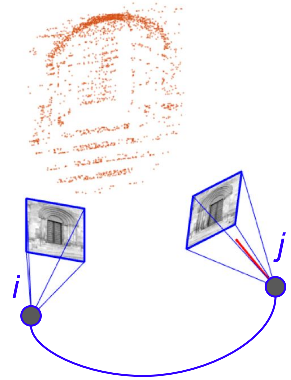

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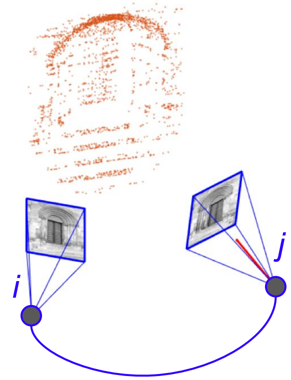

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Equality (to 1st order) if $$M = \frac{\operatorname{tr}(H)}{2}\mathrm{I}_3 - H$$

See paper for details.

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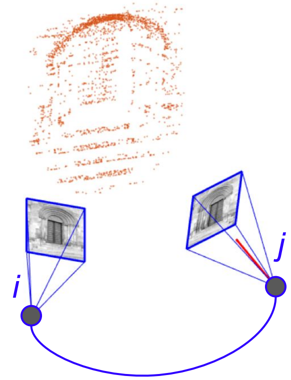

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Equality (to 1st order) if

$$M = \frac{\operatorname{tr}(H)}{2}\mathrm{I}_3 - H$$

← *while H is always p.s.d., M is almost always indefinite*

See paper for details.

New objective

$$\min_{\{R_i \in SO(3)\}} -\operatorname{tr}(\tilde{\mathbf{R}}\mathbf{R}\mathbf{R}^\top) \quad \Rightarrow \quad \min_{\{R_i \in SO(3)\}} -\operatorname{tr}(\mathbf{N}\mathbf{R}\mathbf{R}^\top)$$

$$\tilde{\mathbf{R}} = \begin{pmatrix} 0 & \tilde{R}_{12}^\top & \tilde{R}_{13}^\top & \dots & \tilde{R}_{1n}^\top \\ \tilde{R}_{12} & 0 & \tilde{R}_{23}^\top & \dots & \tilde{R}_{2n}^\top \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{1n} & \tilde{R}_{2n} & \tilde{R}_{3n} & \dots & 0 \end{pmatrix}$$

$$\mathbf{N} = \begin{pmatrix} 0 & M_{12}\tilde{R}_{12}^\top & M_{12}\tilde{R}_{12}^\top & \dots & M_{1n}\tilde{R}_{1n}^\top \\ \tilde{R}_{12}M_{12} & 0 & M_{23}\tilde{R}_{23}^\top & \dots & M_{2n}\tilde{R}_{2n}^\top \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{1n}M_{1n} & \tilde{R}_{2n}M_{2n} & \tilde{R}_{3n}M_{3n} & \dots & 0 \end{pmatrix}$$

SDP-O(3)-ISO:

$$\min_{\mathbf{X} \succeq 0} -\operatorname{tr}(\tilde{\mathbf{R}}\mathbf{X})$$
$$\text{s.t. } \mathbf{X}_{ii} = \mathrm{I}_3$$

$\Rightarrow$

SDP-O(3)-ANISO:

$$\min_{\mathbf{X} \succeq 0} -\operatorname{tr}(\mathbf{N}\mathbf{X})$$
$$\text{s.t. } \mathbf{X}_{ii} = \mathrm{I}_3$$

Using standard relaxation

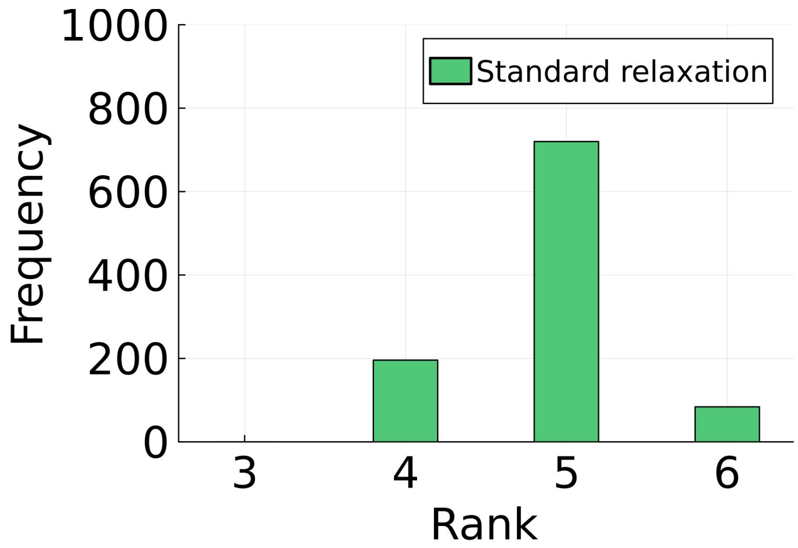
SDP-O(3)-ANISO:

$$\min_{\mathbf{X}\succeq 0} -\operatorname{tr}(\mathbf{N}\mathbf{X})$$

$$\text{s.t. } \mathbf{X}_{ii} = \mathrm{I}_3$$

Doesn’t work with anisotropic cost

Synthetic data experiments



$$f(Q) = -\operatorname{tr}(M\tilde{R}Q^T),\ M \text{ indefinite}$$

- Good approximation on SO(3) and conv(SO(3))
- Yields strictly smaller values on O(3) than on SO(3)

See detailed analysis in the paper.

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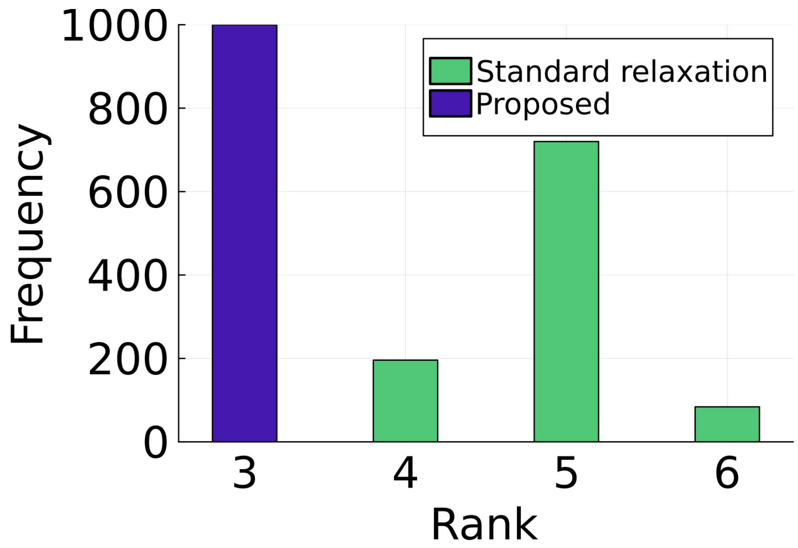

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See detailed analysis in the paper.

Result #2: new convex relaxation

Keep all *SO(3)* constraints:

$$\min_{\{R_i\}} -\operatorname{tr}(\mathbf{N}\mathbf{R}\mathbf{R}^\top)$$
$$\text{s.t. } R_i R_i^\top = \mathrm{I}_3$$
$$R_i R_j^\top \in SO(3)$$

Take Lagrange-dual twice → Linear SDP:

SDP-cSO(3):

$$\min_{\mathbf{X}\succeq 0} -\operatorname{tr}(\mathbf{N}\mathbf{X})$$
$$\text{s.t. } \mathbf{X}_{ii} = \mathrm{I}_3$$
$$\mathbf{X}_{ij} \in \operatorname{convhull}(SO(3))$$

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$$\boxed{\mathbf{X}_{ij} \in \operatorname{convhull}(SO(3))}$$

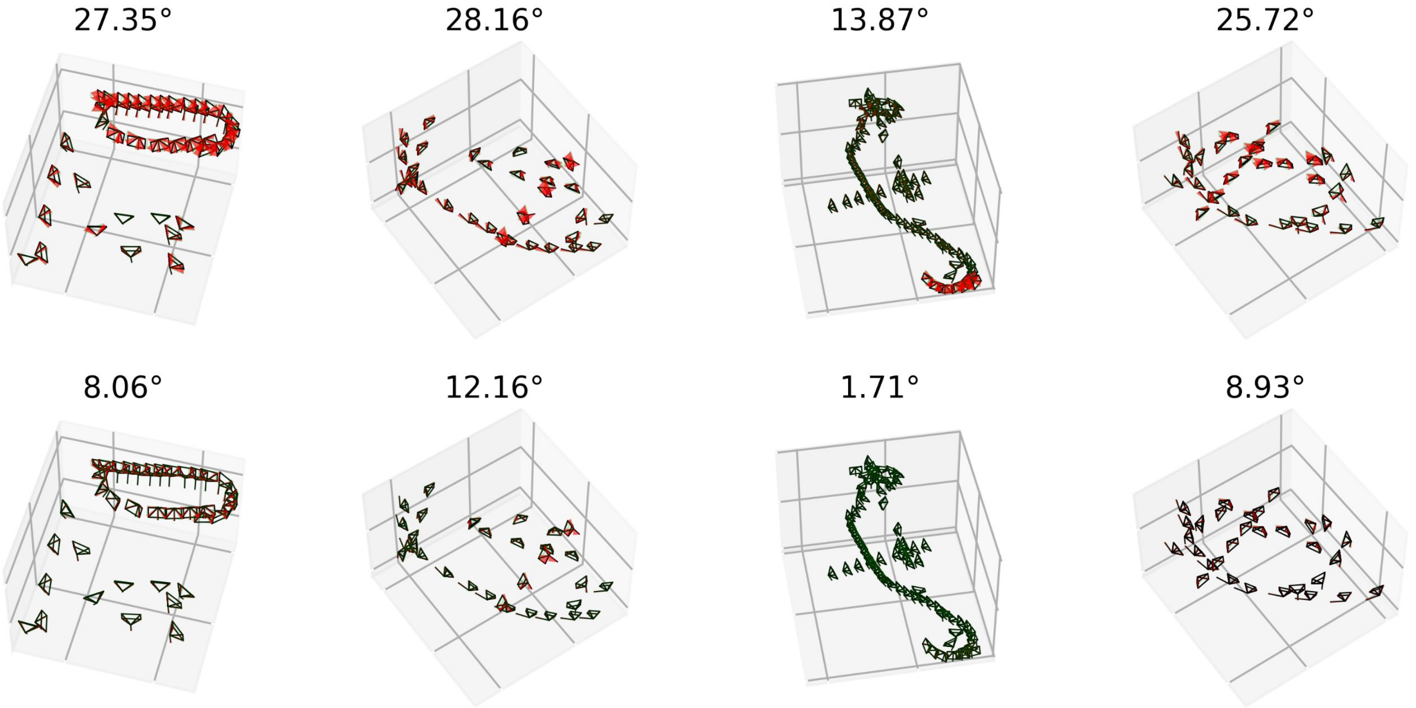
SDP-constraint [Saundersson et al. 2014]

Better reconstructions

RMS angular errors

Standard isotropic SDP-O(3)

Proposed **SDP-cSO(3)**



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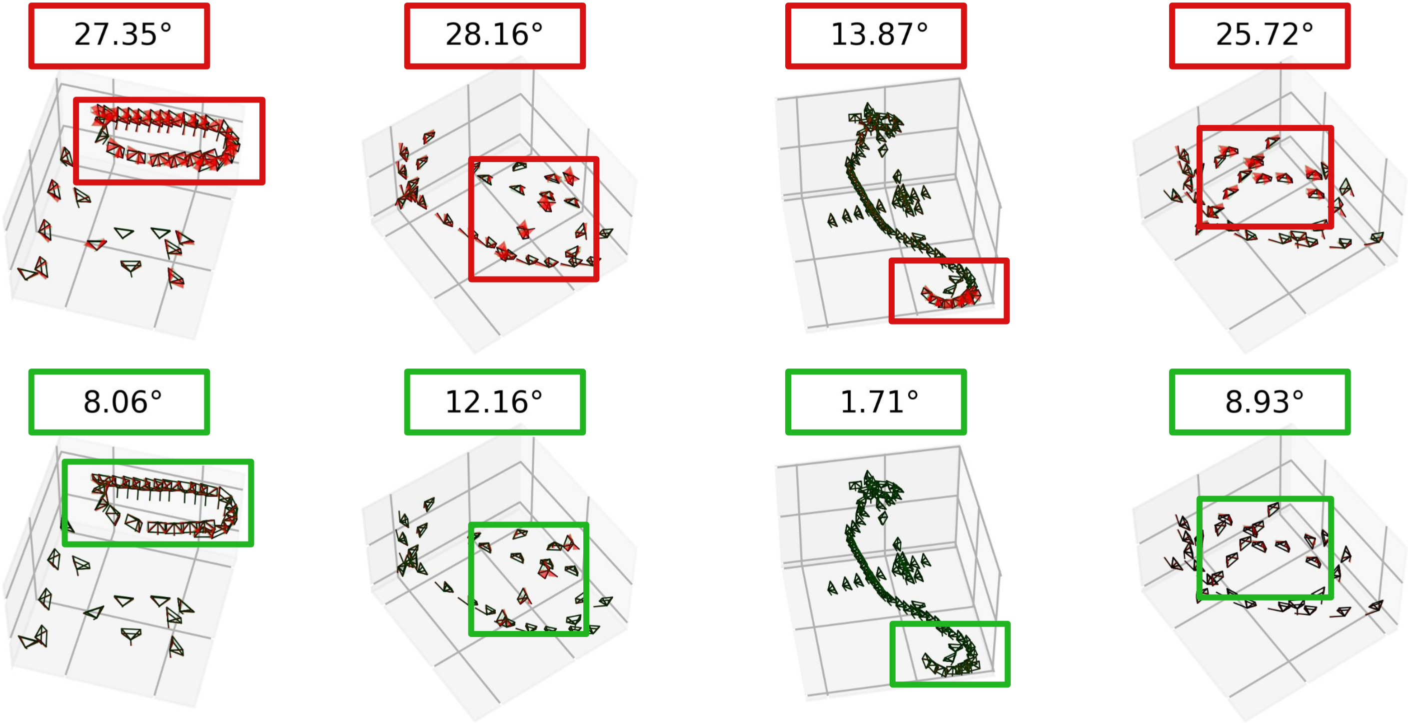
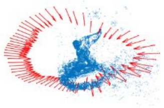
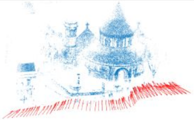
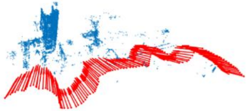
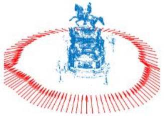
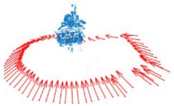


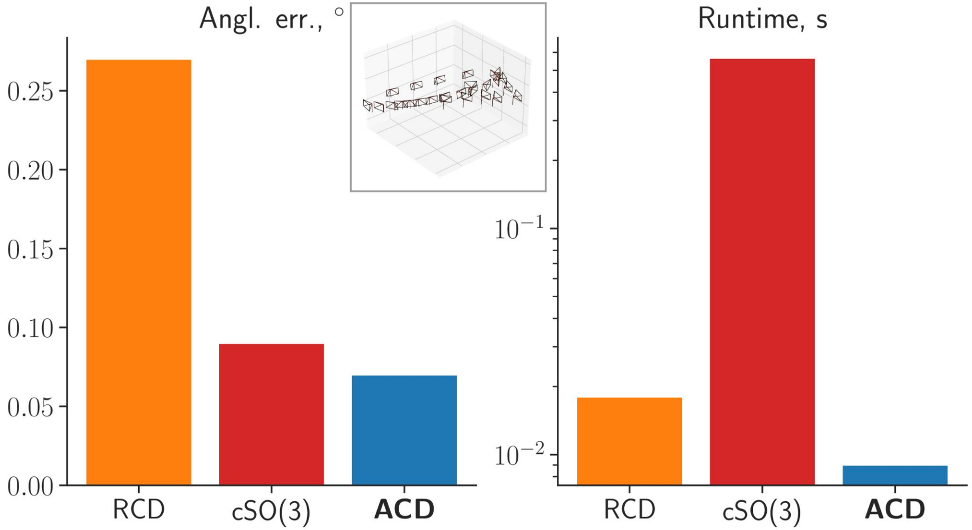
Better reconstructions

Dataset		Method	rank(X^*)	$\sqrt{\sum_i \|R_i - R_i^*\|_F^2}$	Runtime, s
LU Sphinx		SDP-O(3)-ISO	3	0.0944	2
70 cameras		SDP-O(3)-ANISO	7	18.6037	460
85% indef.		**SDP-cSO(3)**	3	**0.0740**	5
Round Church		SDP-O(3)-ISO	3	0.1399	6
92 cameras		SDP-O(3)-ANISO	6	26.3808	632
98% indef.		**SDP-cSO(3)**	3	**0.1267**	55
UWO		SDP-O(3)-ISO	3	0.3142	14
114 cameras		SDP-O(3)-ANISO	6	22.6873	1929
77% indef.		**SDP-cSO(3)**	3	**0.2274**	7
Tsar Nikolai I		SDP-O(3)-ISO	3	0.1170	7
89 cameras		SDP-O(3)-ANISO	6	26.8944	1245
87% indef.		**SDP-cSO(3)**	3	**0.0534**	5
Vercingetorix		SDP-O(3)-ISO	3	0.3146	2
69 cameras		SDP-O(3)-ANISO	6	14.8244	242
77% indef.		**SDP-cSO(3)**	3	**0.2910**	4

See paper for more results.

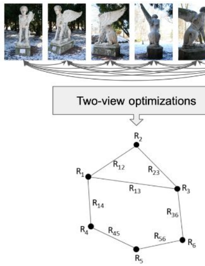
Fast solver?

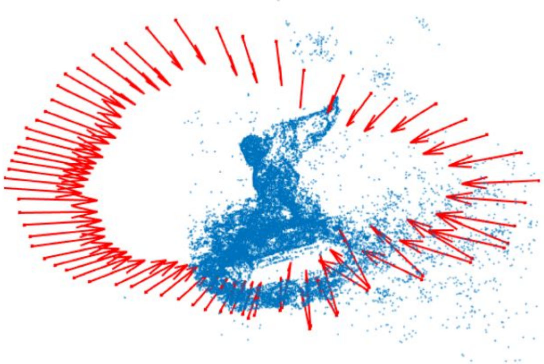
- We used generic SDP solver
- Fast dedicated solver (ACD): arxiv.org/abs/2506.01940

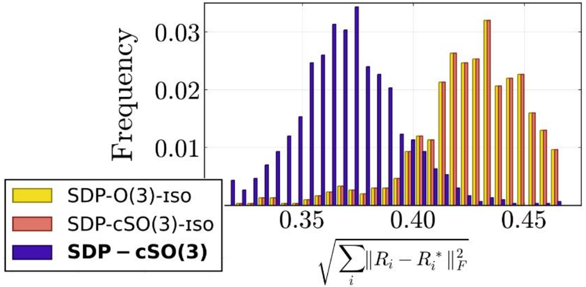


[Lochman et al. Making Rotation Averaging Fast and Robust with Anisotropic Coordinate Descent. BMVC 2025]

Thank you







Dataset	Method	Mahal. err.	Angl. err.
LU Sphinx	SDP-O(3)-ISO	0.388	0.46
	Spectral	0.420	1.17
	SDP-cSO(3)	**0.207**	**0.36**
Round Church	SDP-O(3)-ISO	0.631	0.59
	Spectral	0.437	1.20
	SDP-cSO(3)	**0.368**	**0.54**
UWO	SDP-O(3)-ISO	1.481	1.19
	Spectral	6.125	7.07
	SDP-cSO(3)	**0.727**	**0.86**
Tsar Nikolai I	SDP-O(3)-ISO	0.687	0.48
	Spectral	0.344	0.71
	SDP-cSO(3)	**0.188**	**0.22**
Vercingetorix	SDP-O(3)-ISO	0.431	1.53
	Spectral	30.970	86.94
	SDP-cSO(3)	**0.423**	**1.42**
Eglise Du Dome	SDP-O(3)-ISO	0.224	0.24
	Spectral	**0.119**	0.22
	SDP-cSO(3)	0.188	**0.21**
King's College	SDP-O(3)-ISO	0.229	0.76
	Spectral	0.251	1.00
	SDP-cSO(3)	**0.130**	**0.37**
Kronan	SDP-O(3)-ISO	**0.738**	**0.76**
	Spectral	2.622	4.36
	SDP-cSO(3)	1.111	1.38
Alcatraz	SDP-O(3)-ISO	1.333	0.62
	Spectral	**0.667**	0.80
	SDP-cSO(3)	1.011	**0.45**
Museum Barcelona	SDP-O(3)-ISO	2.710	0.79
	Spectral	16.588	7.35
	SDP-cSO(3)	**1.216**	**0.46**
Temple Singapore	SDP-O(3)-ISO	2.420	0.86
	Spectral	**0.719**	**0.46**
	SDP-cSO(3)	1.076	0.55