

Combinative Matching for Geometric Shape Assembly

Nahyuk Lee^{1*}

Juhong Min^{1,2*}

Junhong Lee¹

Chunghyun Park¹

Minsu Cho^{1,3}

¹ Pohang University of Science and Technology (POSTECH)

² Samsung Research America

³ RLWRLD

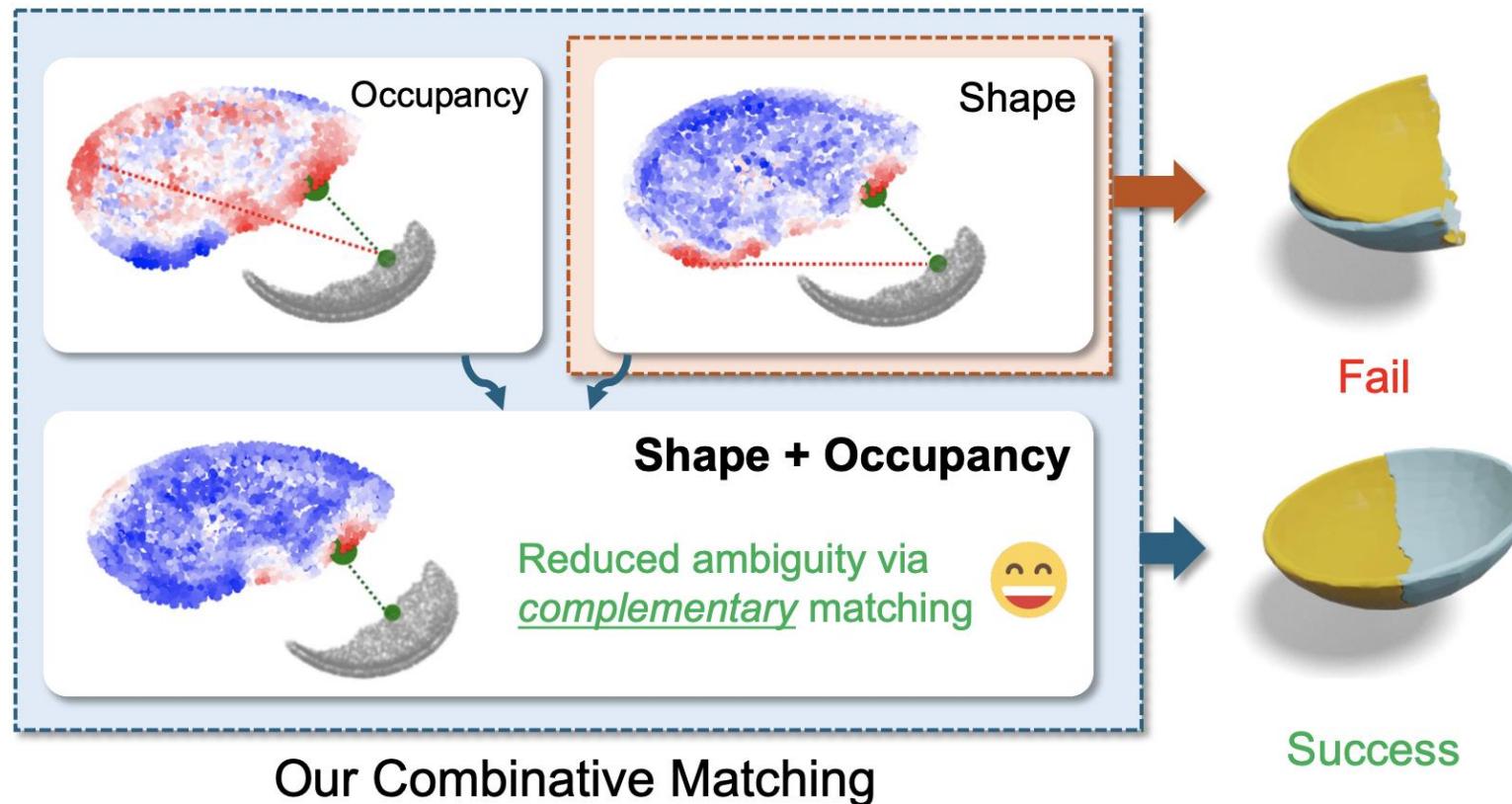
* Equal Contribution



Highlight Presentation 

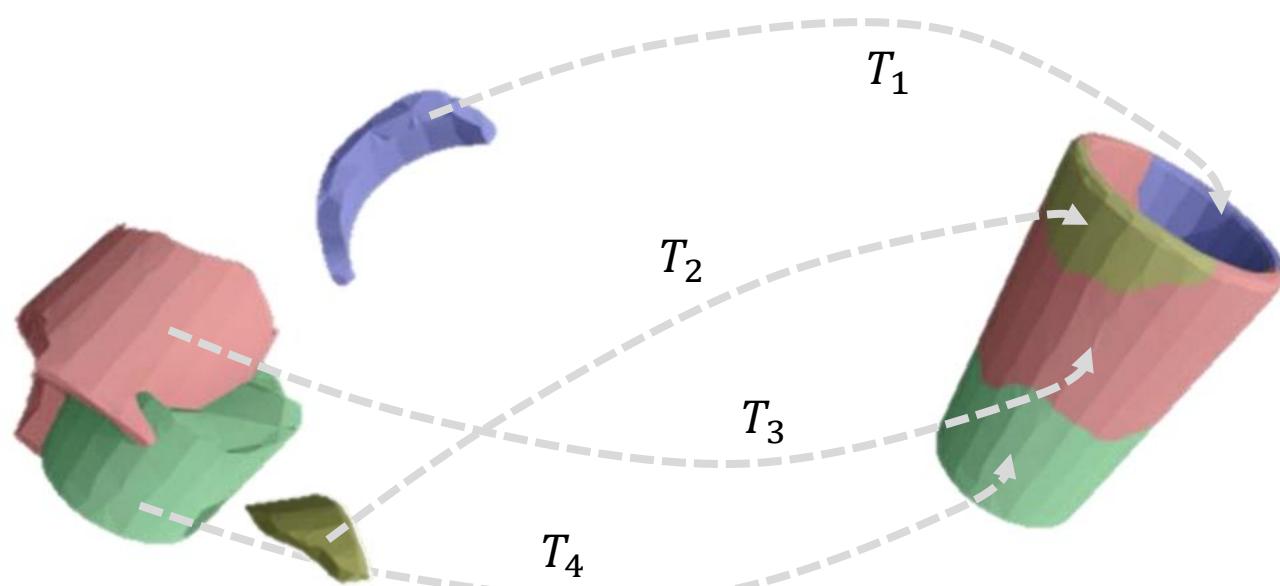


Combinative Matching for Geometric Shape Assembly



3D Geometric Shape Assembly

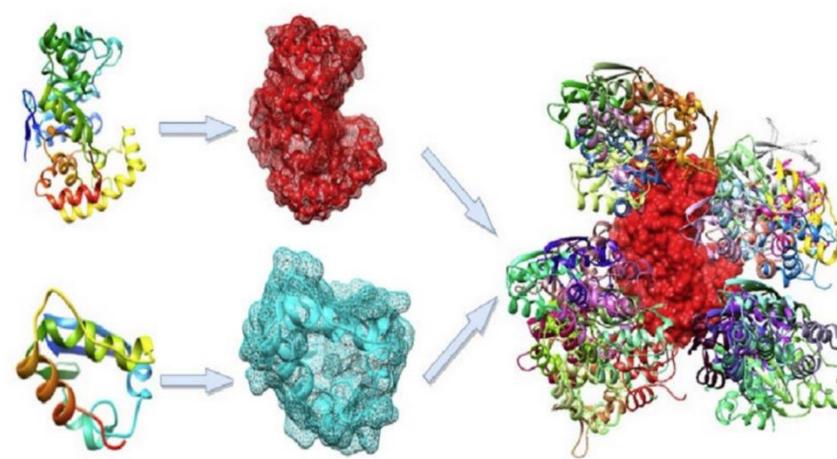
Given a set of fractured pieces $\mathcal{P} = \{P_1, P_2, \dots, P_N\}$, our goal is to recover the 6-DoF pose $\{T_1, T_2, \dots, T_N\}$, in $\text{SE}(3)$ for each piece and restore the underlying object.



3D Geometric Shape Assembly



Archaeology

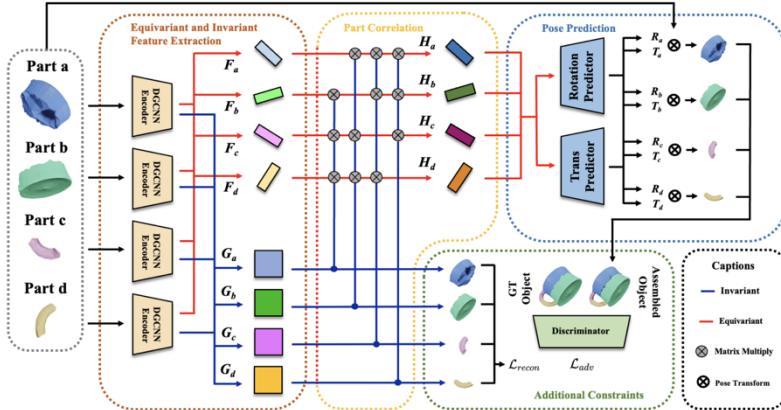


Scientific domain
(e.g., Protein docking)



Robotics

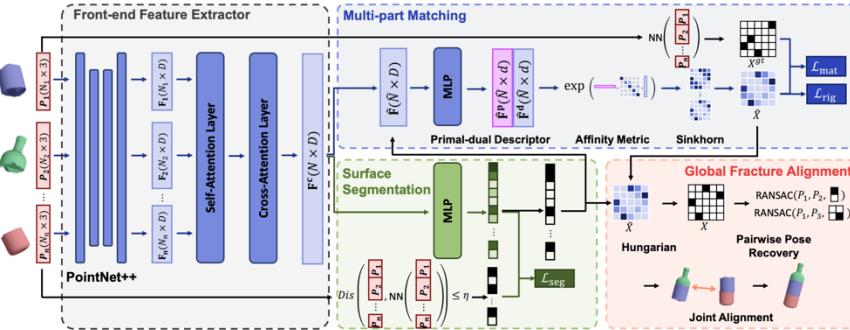
State-of-the-art methods



Wu et al.

(Wu et al., ICCV 2023)

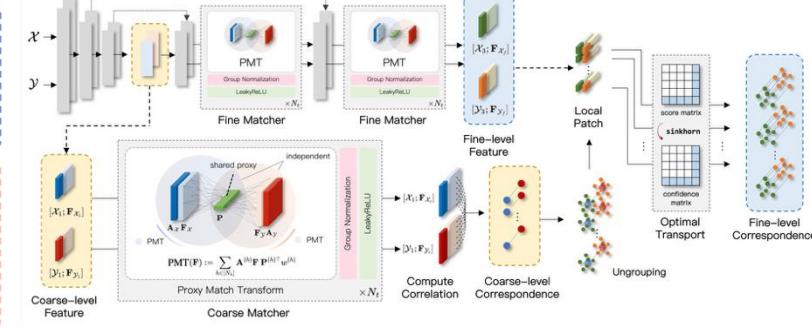
- 😊 Leveraging SE(3) equivariance for multi-part correlations
- 😢 Pose regression with limited spatial information



Jigsaw

(Lu et al., NeurIPS 2023)

- 😊 Effective matching between mating surfaces of parts
- 😢 Heavily rely on segmentation of mating surfaces



PMTR

(Lee et al., ICML 2024)

- 😊 Approximate high-order conv. for dense matching of parts
- 😢 Often fails to establish good correspondences between mating surfaces

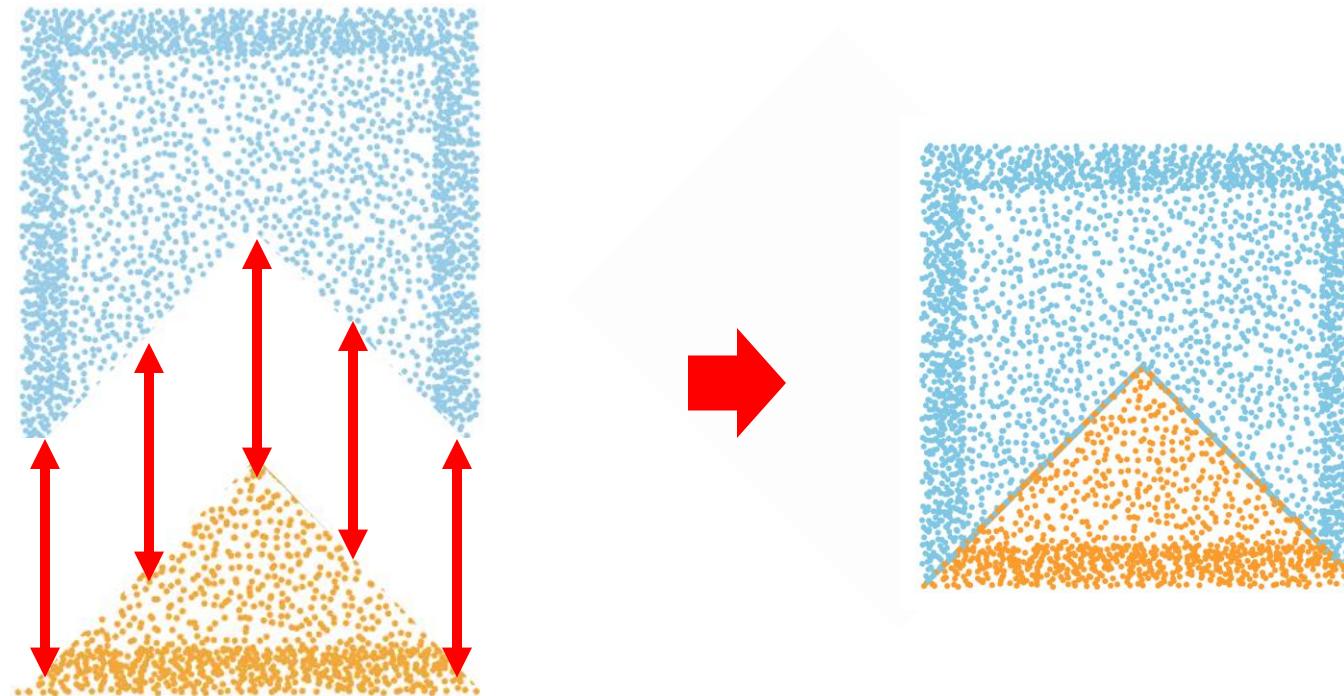
Wu et al., "Leveraging SE(3) Equivariance for Learning 3D Geometric Shape Assembly", ICCV 2023

Lu et al., "Jigsaw: Learning to Assemble Multiple Fractured Objects", NeurIPS 2023

Lee et al., "3D Geometric Shape Assembly via Efficient Point Cloud Matching", ICML 2024

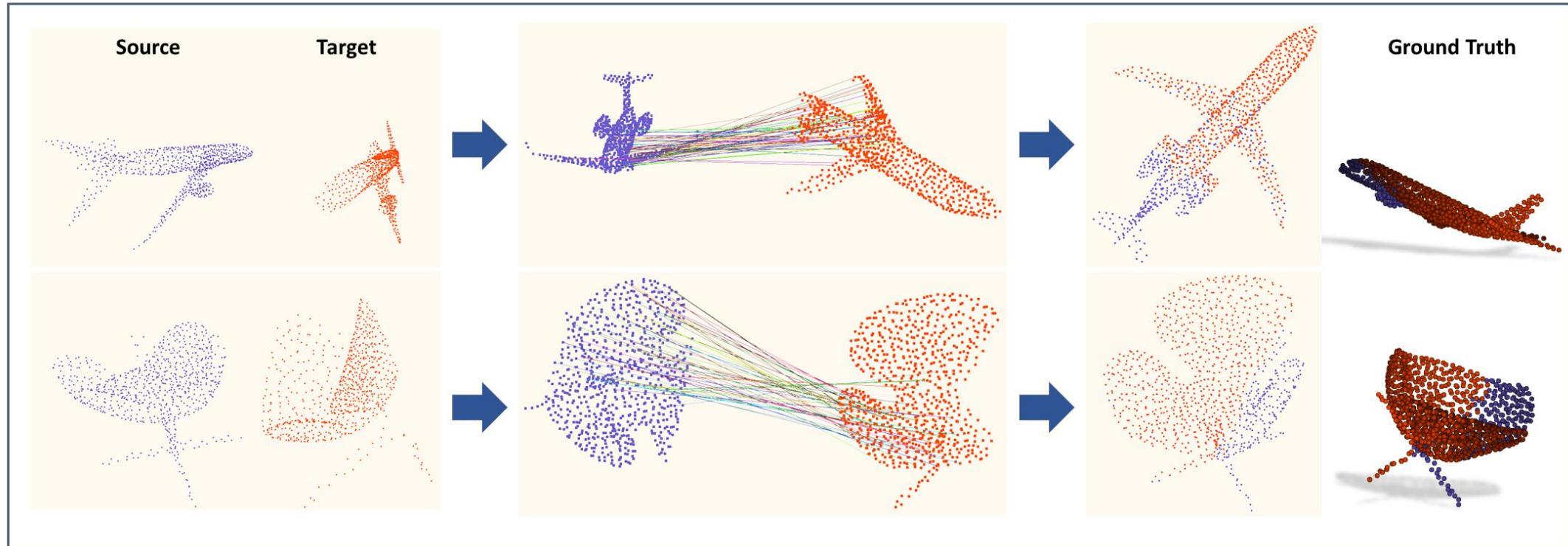
Previous Approach

Equative matching: establishing correspondences based on visual similarity alone
under the assumption of the same appearance between mating parts.



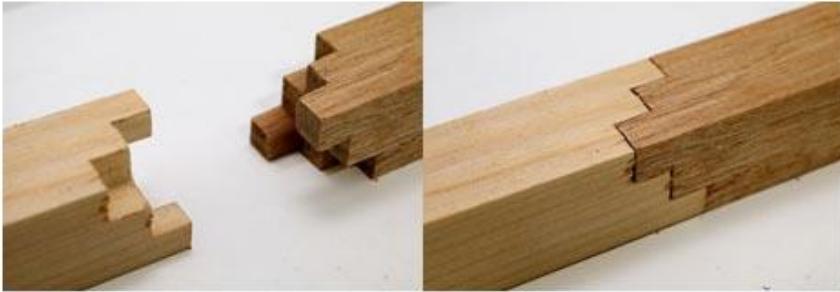
Previous Approach

Equative matching: establishing correspondences based on visual similarity alone
under the assumption of the same appearance between mating parts.

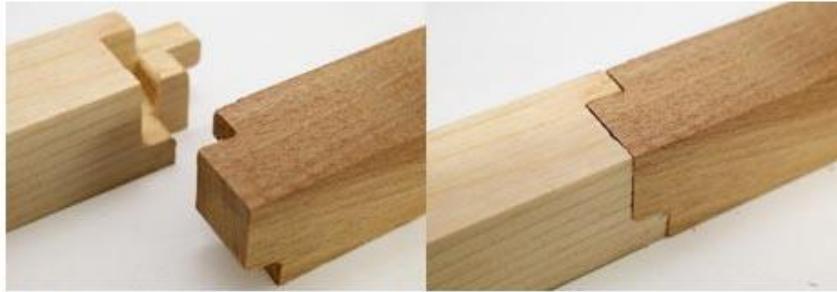


Point Cloud Registration

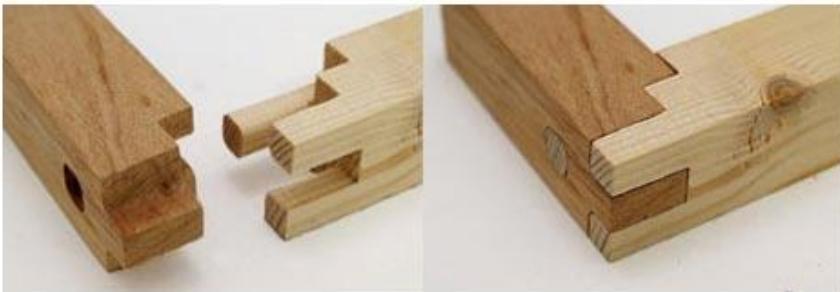
Beyond Equative Matching



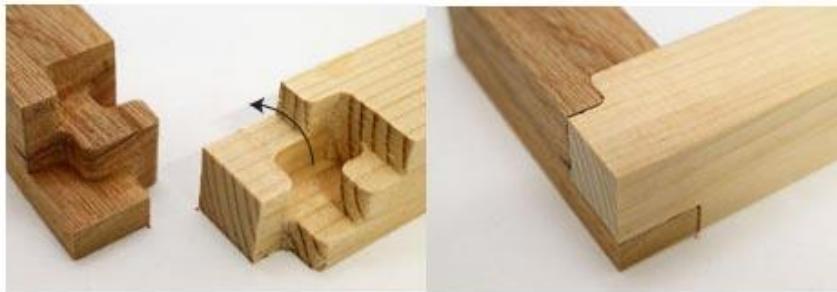
a) I-axial



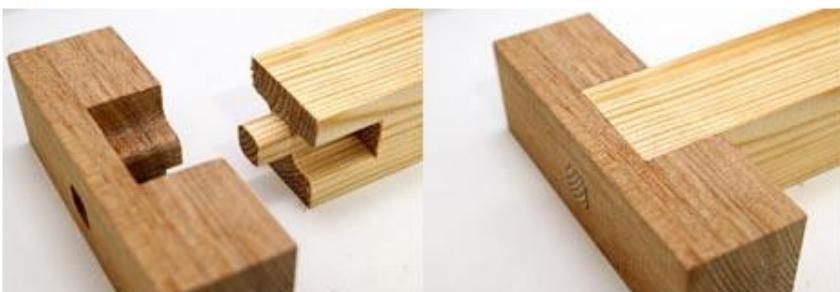
b) I-perpendicular*



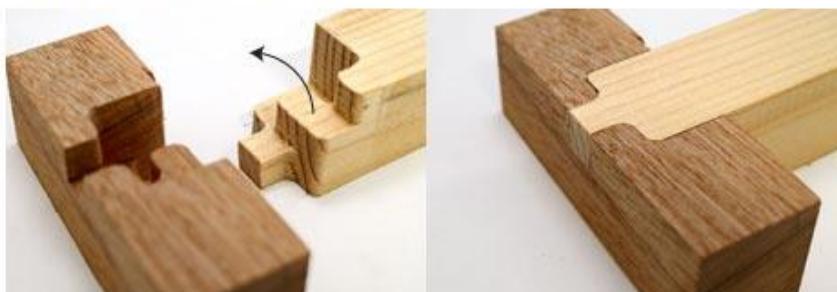
c) L-axial



d) L-perpendicular*

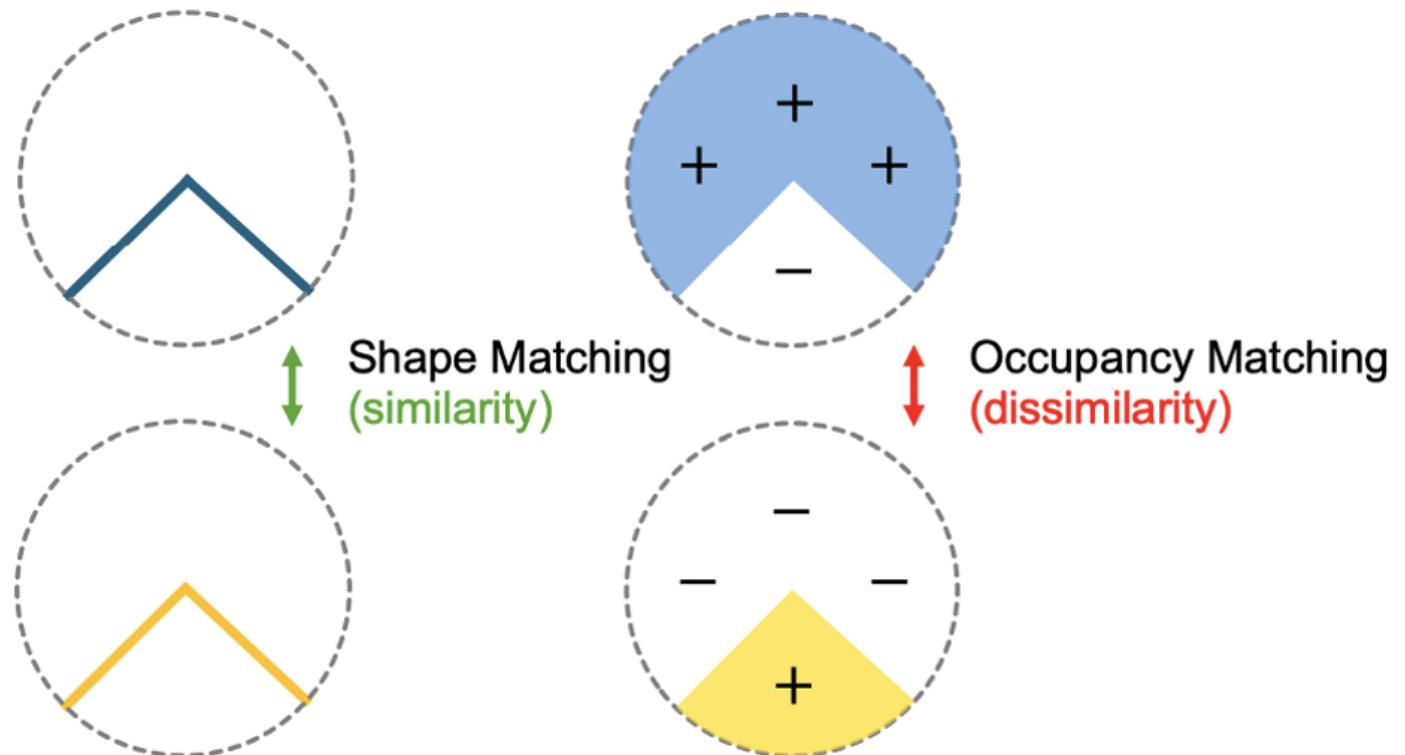
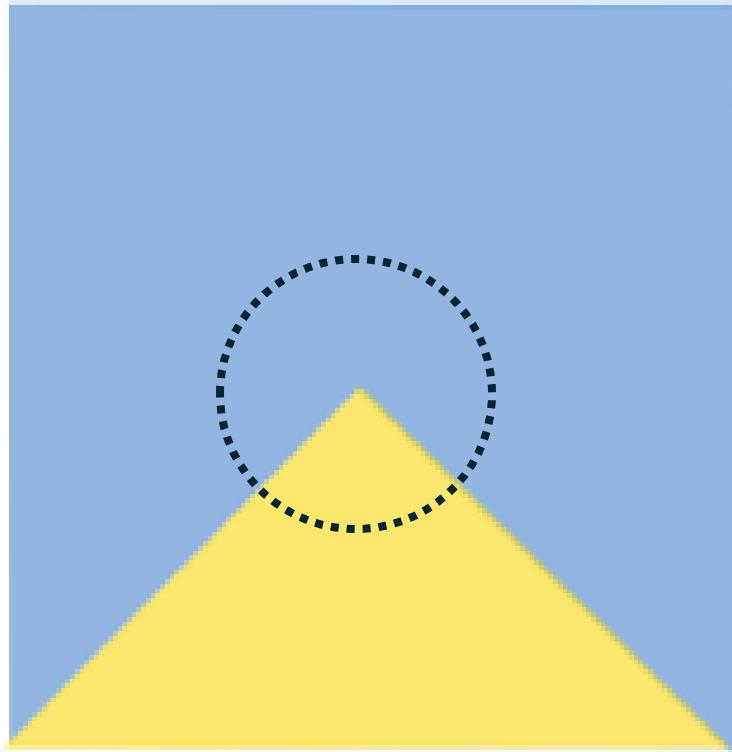


e) T-axial



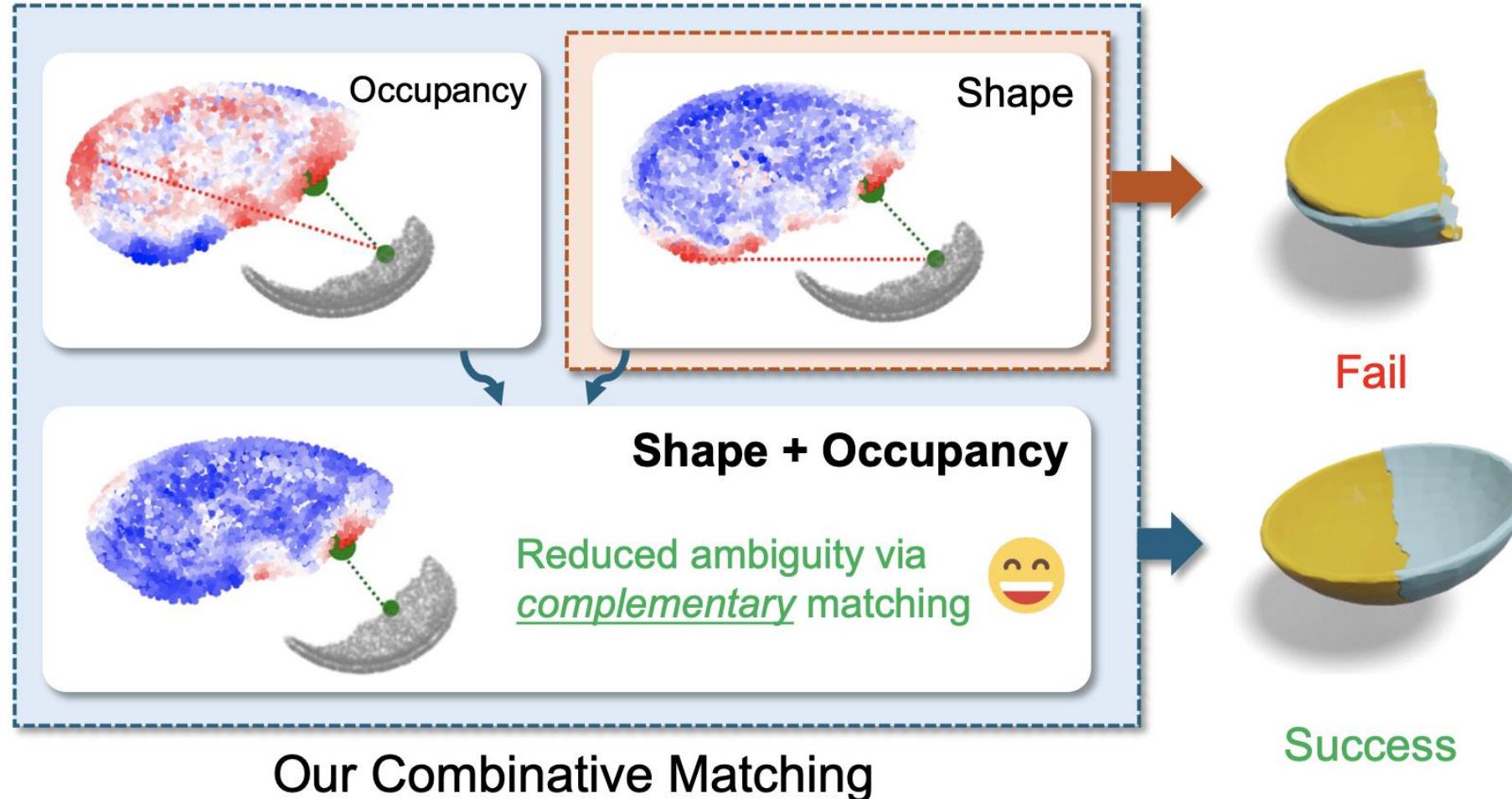
f) T-perpendicular*

Beyond Equiative Matching

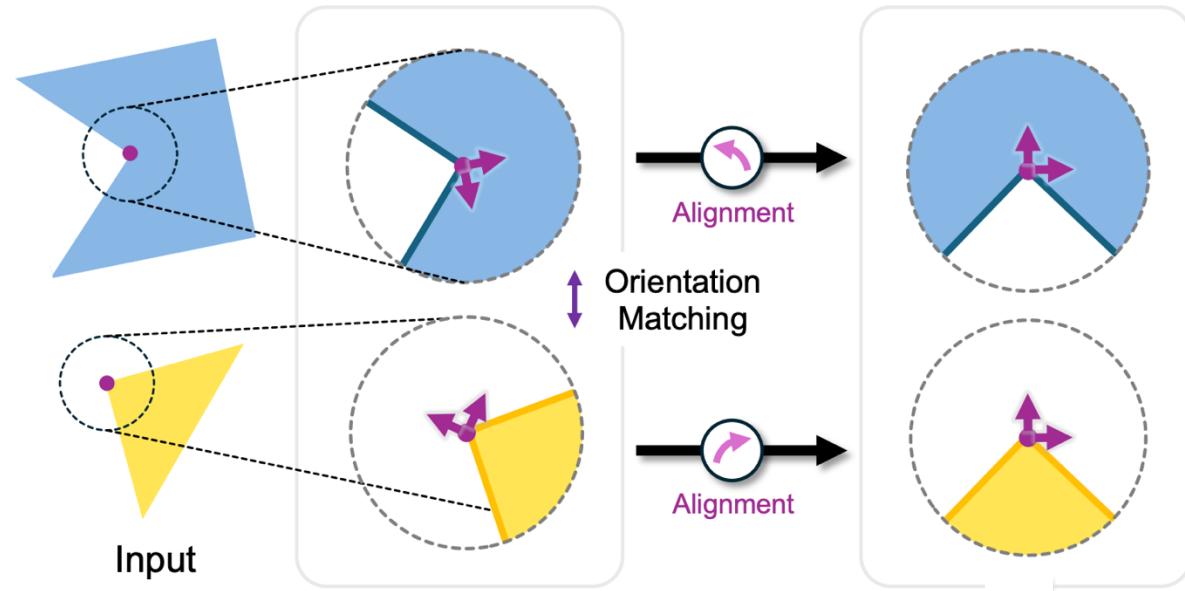


Combinative Matching:

‘combination’ or ‘joining’ of elements form the basis for matching

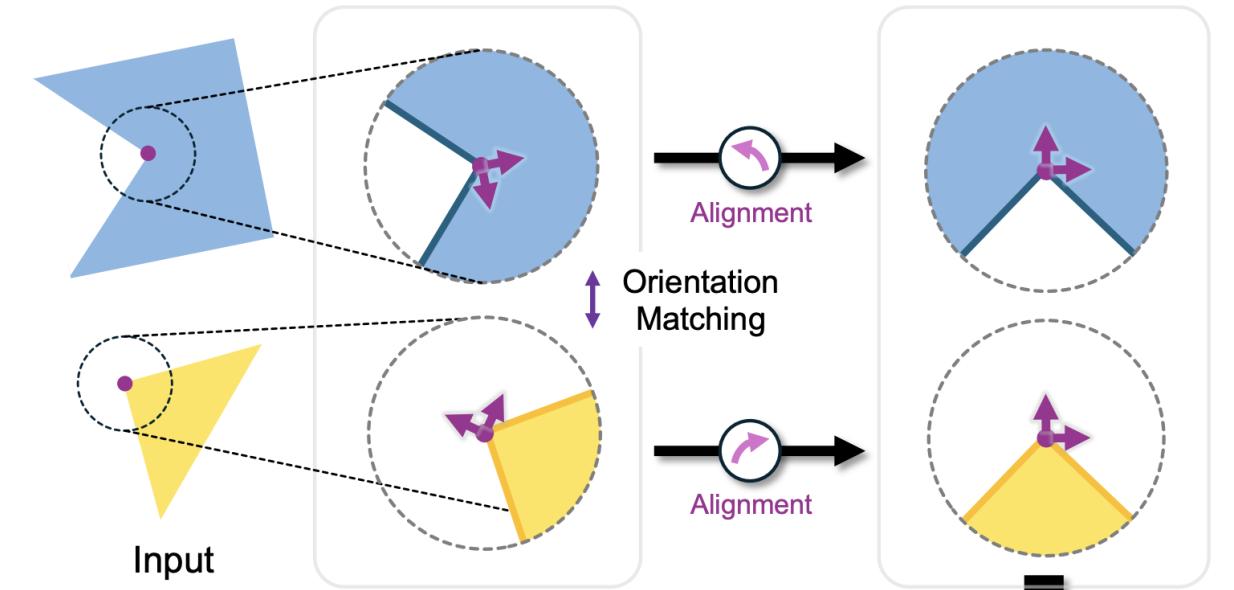


Objectives for Combinative Matching

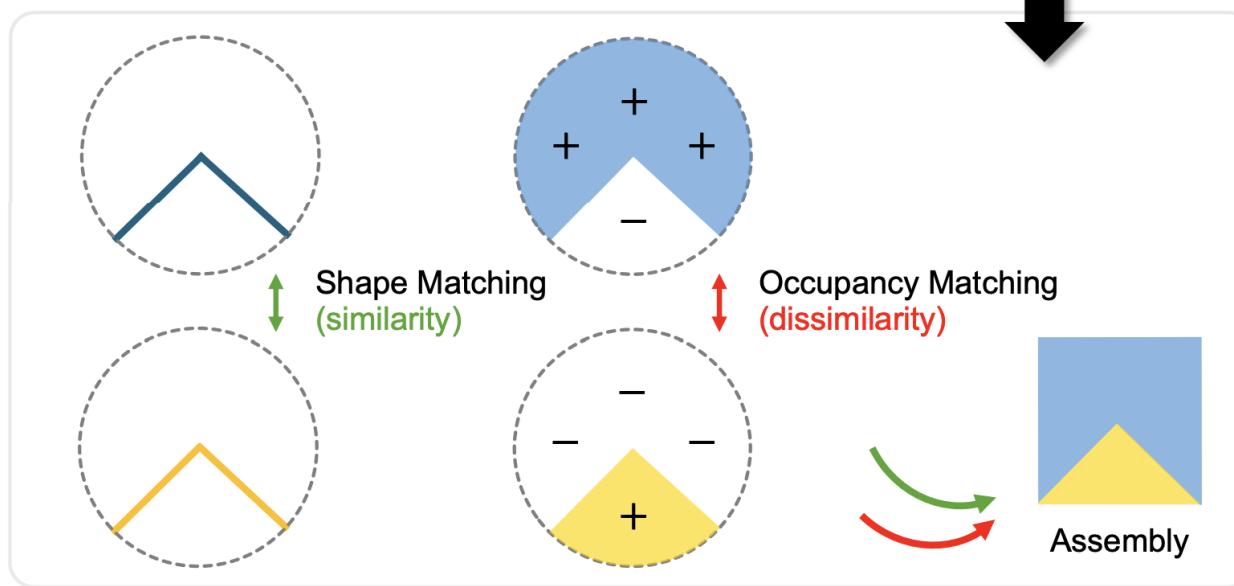


\mathcal{L}_d : maximize 'orientation' consistency
(in $SO(3)$)

Objectives for Combinative Matching



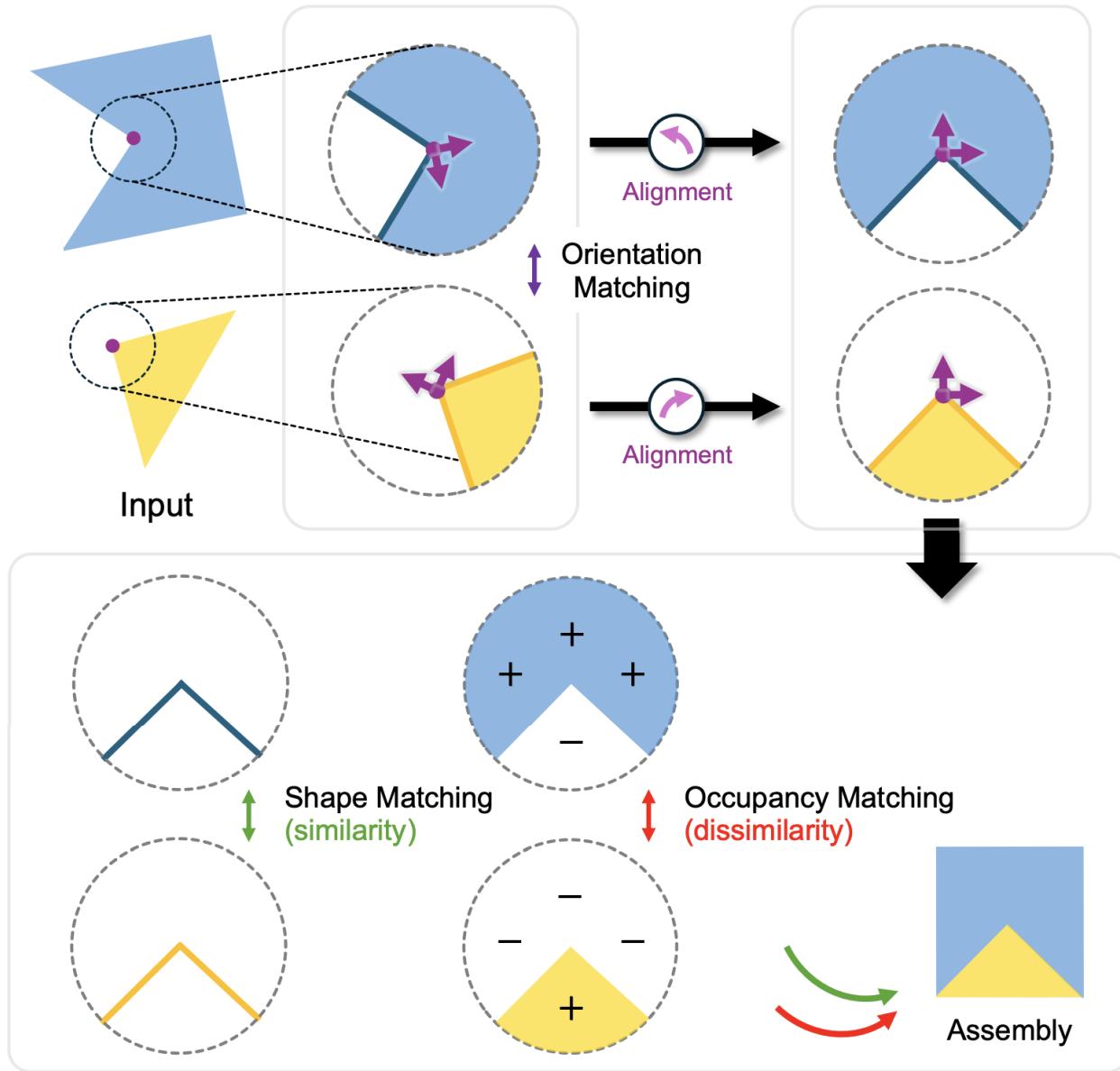
\mathcal{L}_d : maximize 'orientation' consistency
(in $SO(3)$)



\mathcal{L}_s : maximize 'visual' feature similarity

\mathcal{L}_o : maximize 'occupancy' feature dissimilarity

Objectives for Combinative Matching



\mathcal{L}_d : maximize 'orientation' consistency



$$\mathcal{L}_d = \frac{1}{|\mathcal{C}|} \sum_{(i,j) \in \mathcal{C}} \|(\mathbf{F}_d^P)_i \mathbf{R}^P - (\mathbf{F}_d^Q)_j \mathbf{R}^Q\|_F$$

\mathcal{L}_s : maximize 'visual' feature similarity

$$\mathcal{L}_s = \mathbb{E}_{i \sim \mathcal{I}} \left[\log \left(1 + \sum_{j \in \mathcal{E}_p(i)} e^{\alpha_{ij}(d_{ij}^p - \Delta_p)} \cdot \sum_{k \in \mathcal{E}_n(i)} e^{\beta_{ik}(\Delta_n - d_{ik}^n)} \right) \right]$$

$$\alpha_{ij} = \gamma[d_{ij}^p - \Delta_p]_+, \quad \beta_{ik} = \gamma[\Delta_n - d_{ik}^n]_+ \quad d_{ij}^p = \|\hat{\mathbf{F}}_{s,i}^P - \hat{\mathbf{F}}_{s,j}^Q\|_2$$

\mathcal{L}_o : maximize 'occupancy' feature dissimilarity

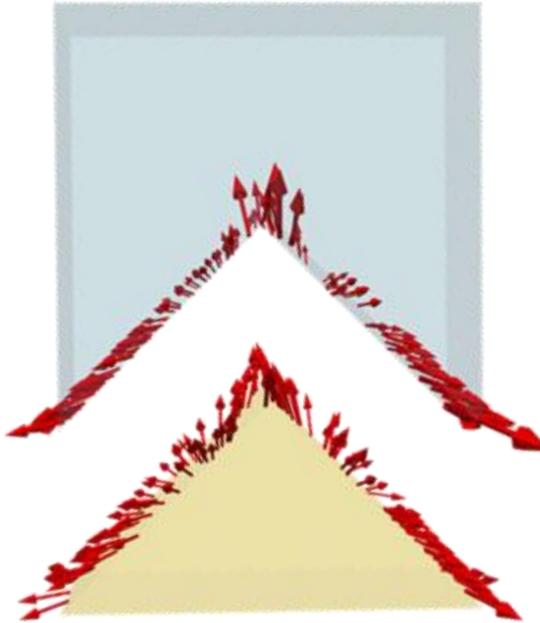
$$\mathcal{L}_o = \mathbb{E}_{i \sim \mathcal{I}} \left[\log \left(1 + \sum_{j \in \mathcal{E}_p(i)} e^{\alpha_{ij}(s_{ij}^p - \Delta_p)} \cdot \sum_{k \in \mathcal{E}_n(i)} e^{\beta_{ik}(\Delta_n - s_{ik}^n)} \right) \right]$$

$$\alpha_{ij} = \gamma[s_{ij}^p - \Delta_p]_+, \quad \beta_{ik} = \gamma[\Delta_n - s_{ik}^n]_+$$

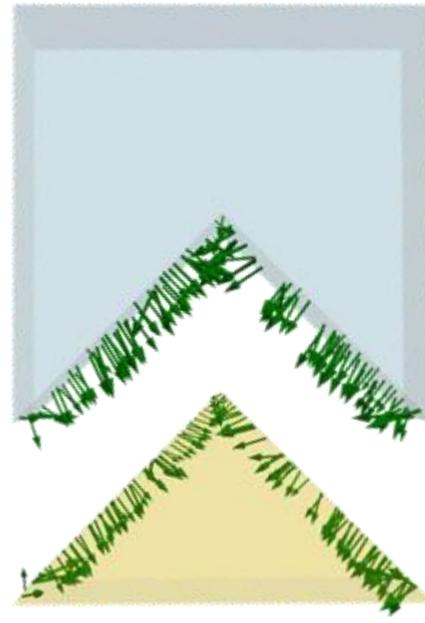
$$s_{ij}^p \approx \cos(\mathbf{F}_{o,i}^P, \mathbf{F}_{o,j}^Q)$$

Learned orientation analysis

Several notable patterns



$$\{\mathbf{x}_i\}_{i=1}^K$$



$$\{\mathbf{y}_i\}_{i=1}^K$$

\mathbf{x}_i : directed toward the center of the surface

\mathbf{x}_i : parallel to the 2D plane of the interface

\mathbf{y}_i : pointing in/outward on concave/convex

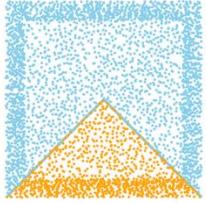
\mathbf{y}_i : magnitudes correlating with the degree of convexity/concavity

src and **trg** orientations are aligned in parallel
(enforced by objective)

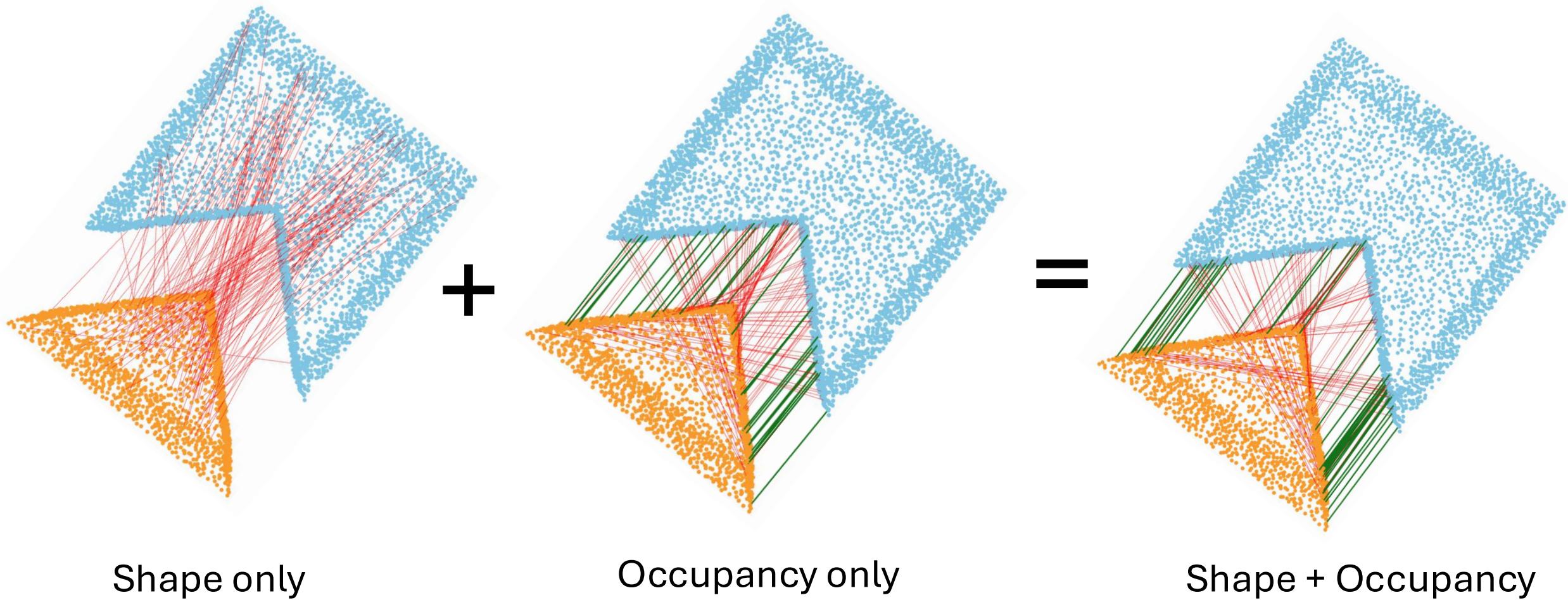
Learned orientations capture surface structures (convex, concave, curvature, 2D plane)
without any explicit supervisions dedicated to these aspects

Learned correlation analysis

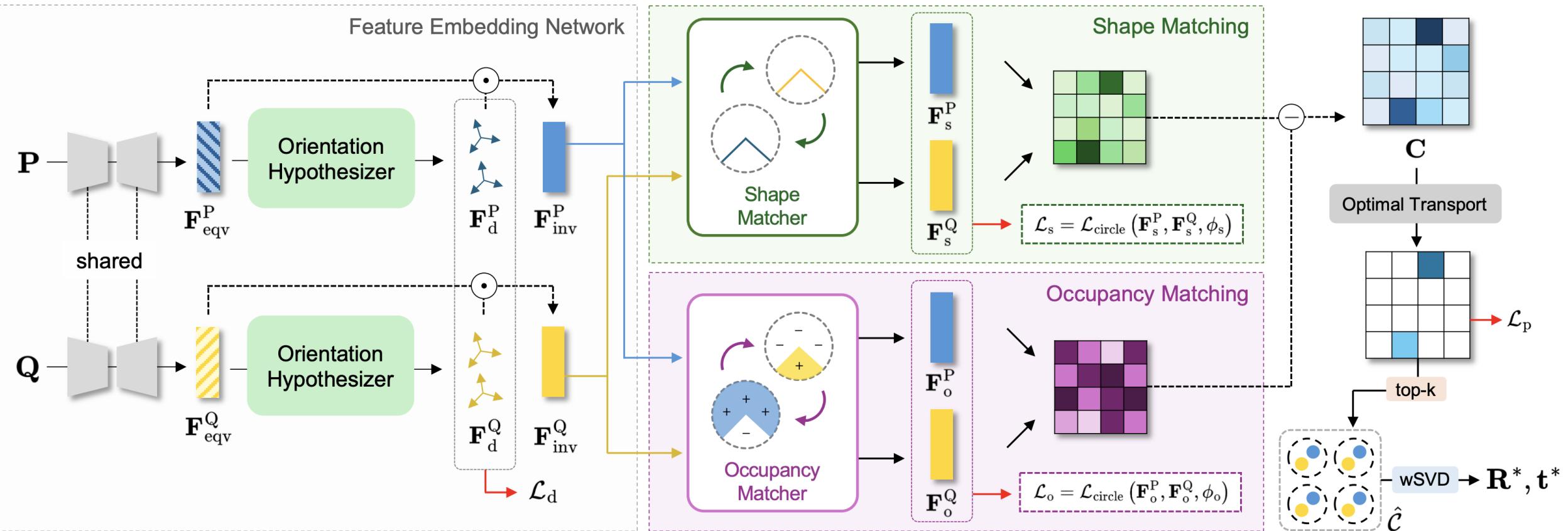
Learning occupancy 'dissimilarity' effectively resolves local ambiguity



Assembly



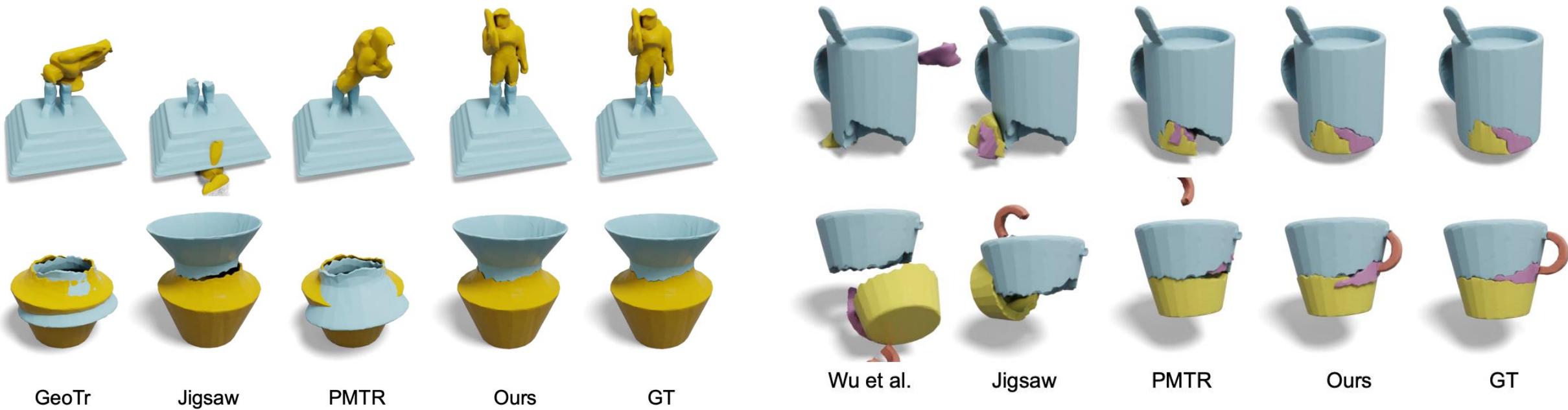
Network Architecture



$$\mathcal{L} = \underbrace{\lambda_s \mathcal{L}_s + \lambda_o \mathcal{L}_o + \lambda_d \mathcal{L}_d}_{\text{Combinative matching objective}} + \mathcal{L}_p \leftarrow \text{Point matching loss}$$

Combinative matching objective

Qualitative Comparisons



Two-part assembly result

Multi-part assembly result

Comparison with SoTA

Method	CRD \downarrow (10^{-2})	CD \downarrow (10^{-3})	RMSE(R) \downarrow ($^{\circ}$)	RMSE(T) \downarrow (10^{-2})
everyday				
NSM [5]	21.71	11.09	83.38	23.71
Wu et al. [48]	20.65	11.66	84.58	22.90
GeoTransformer [34]	0.61	0.51	22.81	7.28
Jigsaw [24]	5.48	1.34	38.73	2.73
PMTR [16]	<u>0.39</u>	<u>0.25</u>	<u>17.14</u>	5.53
CMNet (Ours)	0.28	0.17	12.88	<u>3.78</u>
artifact				
NSM [5]	19.44	6.33	83.22	21.41
Wu et al. [48]	19.17	7.97	85.04	20.90
GeoTransformer [34]	0.89	0.70	33.23	10.30
Jigsaw [24]	6.36	1.45	39.71	3.02
PMTR [16]	<u>0.60</u>	<u>0.42</u>	<u>23.28</u>	7.27
CMNet (Ours)	0.49	0.34	18.77	<u>5.57</u>

Pairwise Shape Assembly

Method	CRD \downarrow (10^{-2})	CD \downarrow (10^{-3})	RMSE(R) \downarrow ($^{\circ}$)	RMSE(T) \downarrow (10^{-2})	PA _{CRD} \uparrow (%)	PA _{CD} \uparrow (%)
everyday						
Global [17, 37]	27.79	15.30	55.42	15.31	36.42	37.90
LSTM [47]	27.69	15.23	54.78	15.24	36.74	38.97
DGL [12]	27.90	13.23	55.76	15.33	36.99	39.70
Wu et al. [48]	28.18	19.70	54.98	15.59	35.66	36.28
Jigsaw [24]	14.13	11.82	41.12	11.74	52.48	60.26
PMTR [16]	<u>6.51</u>	<u>5.56</u>	<u>31.57</u>	<u>9.95</u>	<u>66.95</u>	<u>70.56</u>
CMNet (Ours)	5.18	3.65	27.11	8.13	73.88	77.88
artifact						
Global [17, 37]	26.42	14.92	54.41	14.48	36.67	36.97
LSTM [47]	28.15	14.61	53.59	15.49	36.67	37.25
DGL [12]	27.48	13.91	54.66	15.10	36.66	37.40
Wu et al. [48]	26.02	15.81	54.35	14.27	36.63	37.02
Jigsaw [24]	16.10	9.53	42.01	17.47	56.93	65.58
PMTR [16]	<u>5.67</u>	<u>4.33</u>	<u>31.58</u>	<u>10.08</u>	<u>66.96</u>	<u>71.61</u>
CMNet (Ours)	4.56	3.04	29.21	8.99	71.02	76.32

Multi-part Assembly (Volume-constrained ver.)

Method	RMSE(R) \downarrow ($^{\circ}$)	RMSE(T) \downarrow (10^{-2})	PA _{CD} \uparrow (%)	CD \downarrow (10^{-3})
everyday				
Global [17, 37]	80.7	15.1	24.6	14.6
LSTM [47]	84.2	16.2	22.7	15.8
DGL [12]	79.4	15.0	31.0	14.3
Wu et al. [48]	79.3	16.9	8.41	28.5
DiffAssemble [36]	73.3	14.8	27.5	-
Jigsaw [24]	42.3	10.7	57.3	13.3
PuzzleFusion++ [46]	<u>38.1</u>	8.0	<u>71.0</u>	<u>6.0</u>
CMNet (Ours)	32.0	<u>9.6</u>	77.3	3.5
everyday \rightarrow artifact				
Jigsaw [24]	52.4	22.2	45.6	14.3
PuzzleFusion++ [46]	52.1	13.9	49.6	14.5
CMNet (Ours)	46.0	<u>14.3</u>	52.6	9.8

Multi-part Assembly (Vanilla ver.)

Summary

- Assembly as a more realistic and challenging generative task is key to interacting AI with reality, e.g., manufacturing and design.
- Leveraging **geometric equivariance and invariance** in a careful design is crucial for addressing the tasks in a realistic environment.
- **Combinative matching**, which considers both shape and occupancy relations in matching, significantly improves the performance of geometric assembly and potentially extends to other relational tasks.

