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HONOLULU HAWAII

DAA*: Deep Angular A Star for Image-based Path Planning

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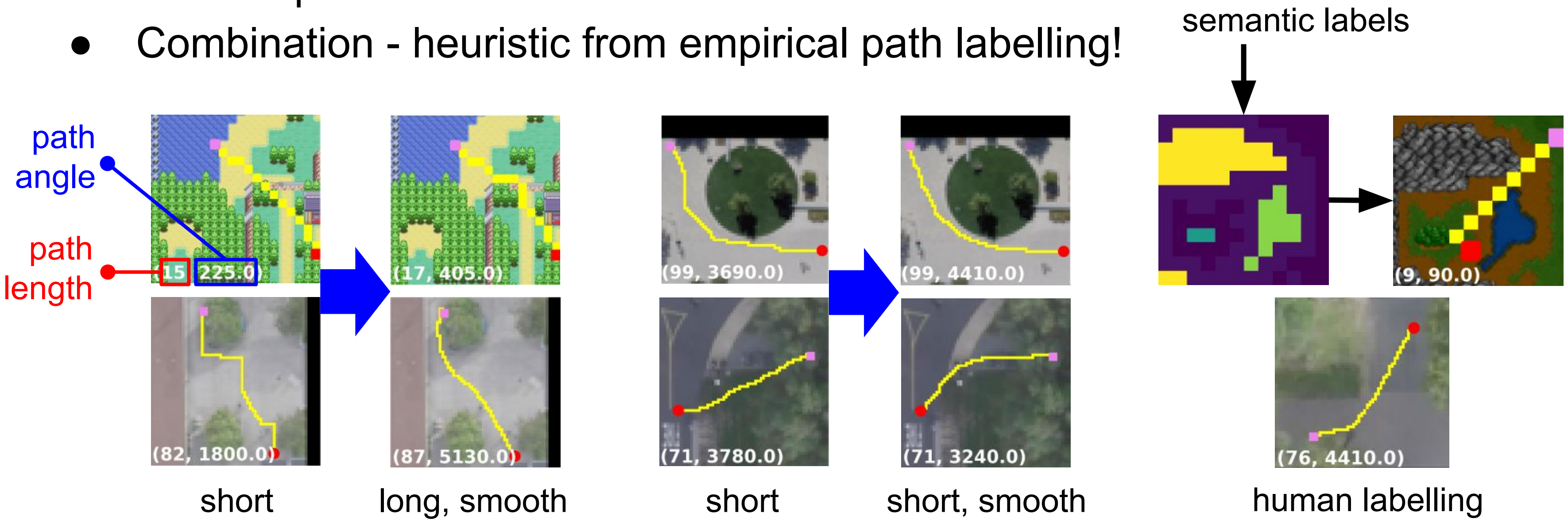
https://github.com/zwxu064/DAAStar.git

Motivation

Path planning on 2D image maps

- Short path?
- Smooth path?
- Combination - heuristic from empirical path labelling!



Motivation

Path Smoothness is as Important as Short Path Length!

Background

- Traditional method
  - Dijkstra’s algorithm (“blind” search) shortest but inefficient
  - A* (use a heuristic function over Dijkstra’s algorithm) imperfect heuristic
  - Theta* (any-angle, true shortest path) shorter, smoother but non-grid, piecewise linear
- Learning-based method
  - Neural A* (supervised learning from path labelling) not smoothness-aware
  - Random-walk (explore reasonable node expansion) inefficient, high uncertainty
  - TransPath (supervised learning from path probability map + short path search) expensive labelling for probability map, not end-to-end

Background

Consider both path shortening and smoothing.

End-to-end learning with more accessible path labelling.

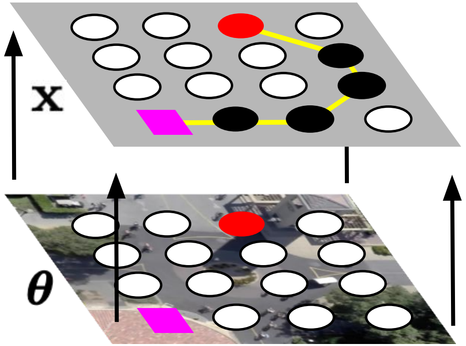
General Optimization Objective

$$\min_{\mathbf{x} \in \mathcal{B}^S} \boldsymbol{\theta}^\top \mathbf{x}\,, \qquad \mathcal{B}^S = \{0,1\}^S$$

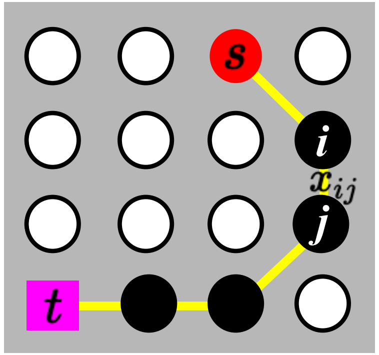
$$\text{subject to} \quad g(i, \mathbf{x}) = \begin{cases} 2, & \forall i \notin \{s, t\} \\ 1, & \forall i \in \{s, t\} \end{cases}$$

each labelled node has two connecting edges or nodes

binary path labelling



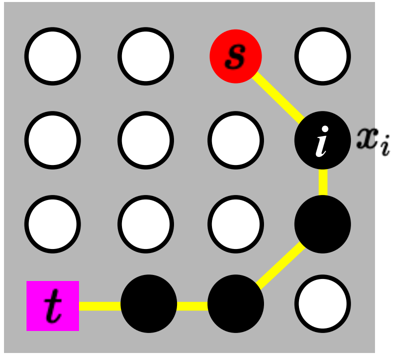
map with a pair of start-target nodes



label on edges

$g(i, \mathbf{x}) = \sum_{j \in \mathcal{N}_i} x_{ij}$

OR



label on nodes

$g(i, \mathbf{x}) = \sum_{j \in \mathcal{N}_i} x_j$

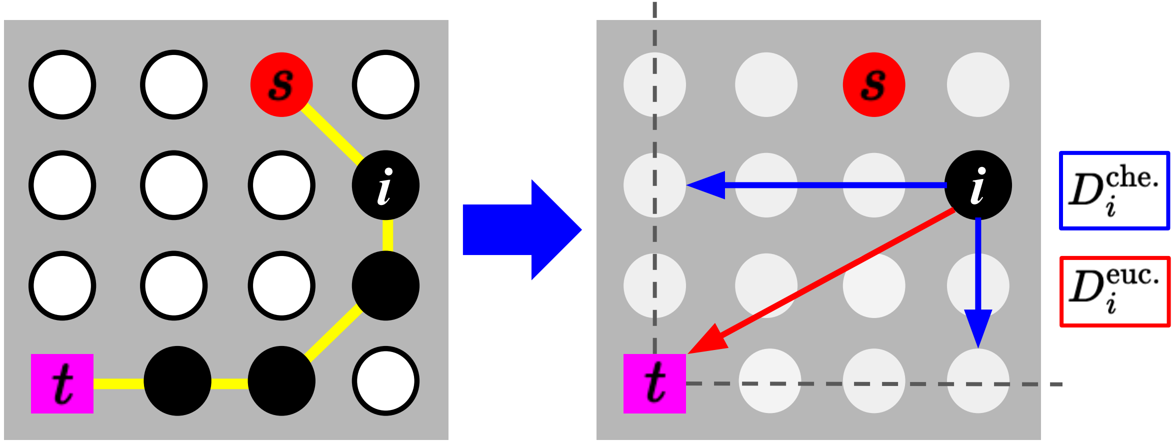
General Optimization Objective

Optimization on nodes

$$\min_{\mathbf{x}\in\mathcal{B}^N} \sum_{i\in\mathcal{V}/\{s,t\}} (\theta_i + \lambda D_i)\, x_i$$

$$\text{subject to} \quad ||\mathcal{N}_i||_0 = \boxed{2,}\ \forall i \in \mathcal{V}/\{s,t\}$$

each labelled node has two connecting nodes ●

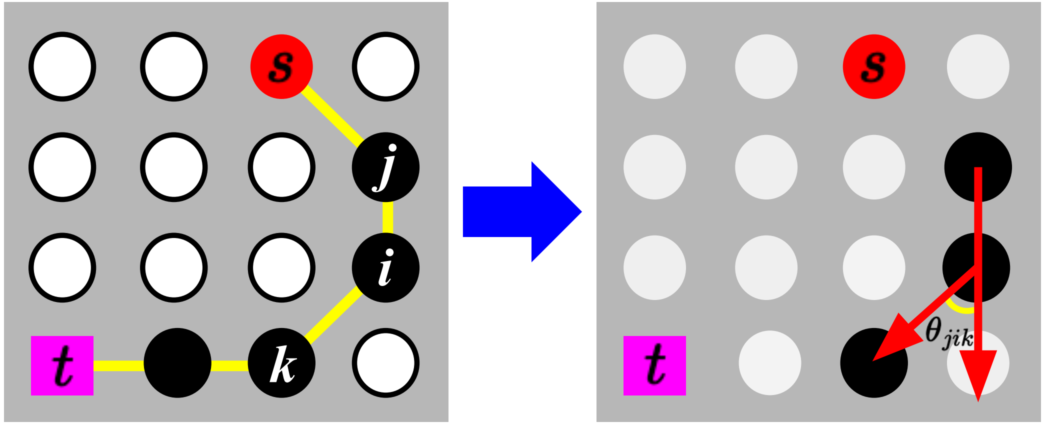


Deep Angular A*

$$\theta_{jik} = \arccos \frac{e_{ji}^{\top} e_{ik}}{||e_{ji}|| \, ||e_{ik}||}$$

$$\min_{\mathbf{x} \in \mathcal{B}^N} \sum_{i \in \mathcal{V}/\{s,t\}} (\theta_i + \lambda D_i)\, x_i + \beta \sum_{(j,k) \in \mathcal{N}_i} \theta_{jik}\, x_j x_i x_k$$

$$\text{subject to} \quad ||\mathcal{N}_i||_0 = 2, \ \forall i \in \mathcal{V}/\{s,t\}$$



Deep Angular A*

However, minimizing over the path angles will easily lead to linear but non-smooth path segments.

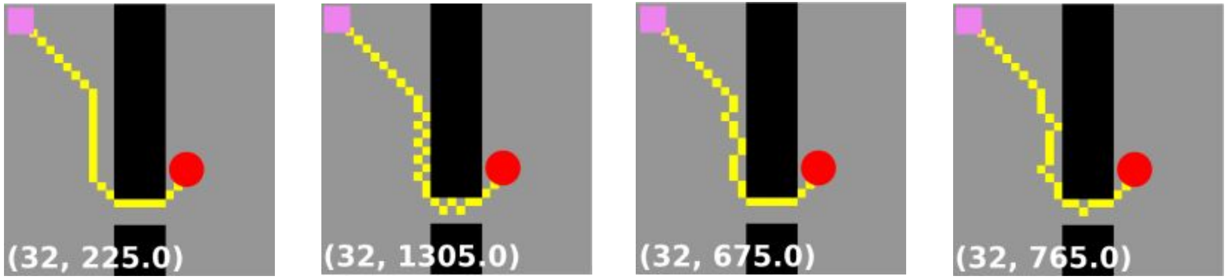
Deep Angular A*

Path angular freedom: smooth path via learning min-max angle adaptation

$$\boxed{h_{jik} = \alpha\theta_{jik} + (1-\alpha)(\pi - \theta_{jik})}$$

$$\min_{\mathbf{x}\in\mathcal{B}^N} \sum_{i\in\mathcal{V}/\{s,t\}} (\theta_i + \lambda D_i)\, x_i + \beta \sum_{(j,k)\in\mathcal{N}_i} \boxed{h_{jik}}\, x_j x_i x_k$$

$$\text{subject to} \quad ||\mathcal{N}_i||_0 = 2,\ \forall i \in \mathcal{V}/\{s,t\}$$



$\alpha = 1$ DAA*-min | $\alpha = 0$ DAA*-max | $\alpha \in (0,1)$ DAA*-mix | reference from policy decision

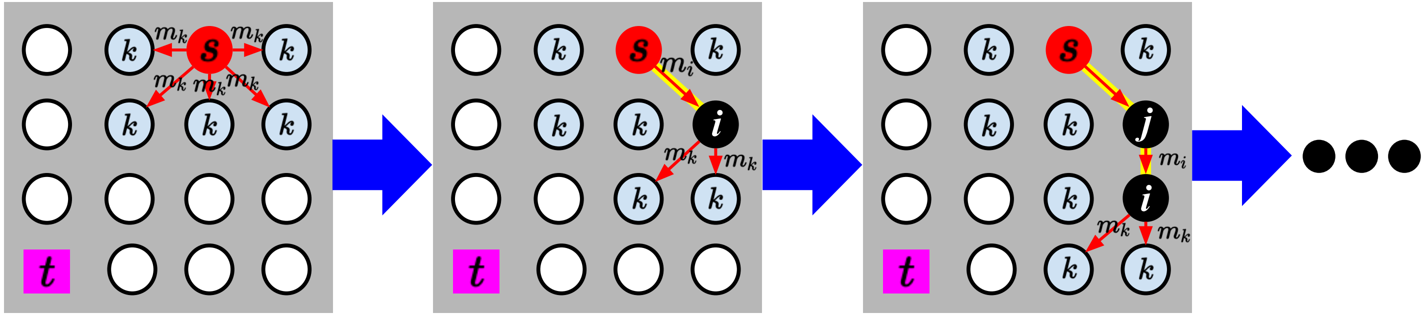
Deep Angular A*

Initial m_k on all nodes are $+\infty$

$$m_k \xleftarrow{\text{update}} \begin{cases} \min(m_k,\ \theta_s), & k \in \mathcal{N}_s \\ \min(m_k,\ \theta_i + m_i + \kappa h_{jik}), & k \in \mathcal{V}/\{s \cup \mathcal{N}_s\} \end{cases}$$

$$c_k = \lambda(\theta_k + D_k) + (1-\lambda) m_k$$

$m_k = \theta_s$ $\qquad$ $m_k = \theta_i + m_i + \kappa h_{sik}$ $\qquad$ $m_k = \theta_i + m_i + \kappa h_{jik}$



$i = \arg\min c_k$ $\qquad$ $j \leftarrow i,\ i = \arg\min c_k$

Experiment

Dataset and path labelling

	Train	Val	Test	Graph
MPD	800	100 (×6)	100 (×15)	32×32
TMPD	3,200	400 (×6)	400 (×15)	64×64
CSM	3,200	400 (×6)	400 (×15)	64×64
Warcraft	2,500 (×4)	250 (×4)	250 (×4)	12×12
Pokémon	750 (×4)	125 (×4)	125 (×4)	20×20
SDD-intra	6,847	1,478	1,478	64×64
SDD-inter	7,284	1,040	1,040	64×64
Aug-TMPD	512,000	64,000	64,000	64×64

binary map, reference from empirical policy

video-game map, Dijkstra's algorithm on semantic map

drone-view map, human labelling

binary map, Dijkstra's algorithm on probability map

Diverse

Experiment

Evaluation metric

shortest path length

$$\text{SPR:} \quad \frac{1}{K}\sum_{i=1}^{K} \mathbb{1}\left[||\mathcal{P}_i||_0 \le ||\hat{\mathcal{P}}_i||_0\right]$$

path similarity with reference path

$$\text{CD:} \quad \frac{1}{K}\sum_{i=1}^{K}\Big(\sum_{x\in\mathcal{I}_i} \min_{y\in\hat{\mathcal{I}}_i} ||x-y||_2^2 + \sum_{y\in\hat{\mathcal{I}}_i} \min_{x\in\mathcal{I}_i} ||x-y||_2^2\Big)$$

$$\text{PSIM:} \quad 1 - \frac{1}{K}\sum_{i=1}^{K} \min\Big(\frac{||\mathcal{P}_i - \hat{\mathcal{P}}_i||_0}{2\,||\hat{\mathcal{P}}_i||_0}, 1\Big)$$

$$\text{ASIM:} \quad 1 - \frac{1}{K}\sum_{i=1}^{K} \frac{\mathtt{Area}(\mathcal{P}_{ij}, \hat{\mathcal{P}}_i)}{\cup_{j\in\mathcal{J}}\mathtt{Area}(\mathcal{P}_{ij}, \hat{\mathcal{P}}_i)}$$

$$\text{Hist:} \quad \frac{1}{K}\sum_{i=1}^{K} \frac{||\mathcal{M}_i^c||_0}{HW}$$

$$\text{Ep:} \quad \frac{1}{K}\sum_{i=1}^{K} \frac{\max(||\hat{\mathcal{M}}_i^c||_0 - ||\mathcal{M}_i^c||_0, 0)}{||\hat{\mathcal{M}}_i^c||_0}$$

path search efficiency

Experiment

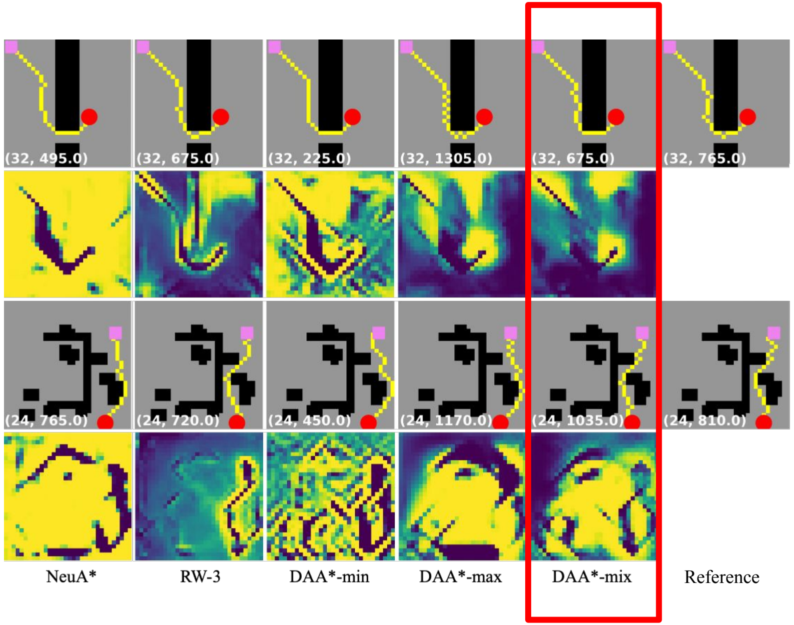
Evaluation on binary maps

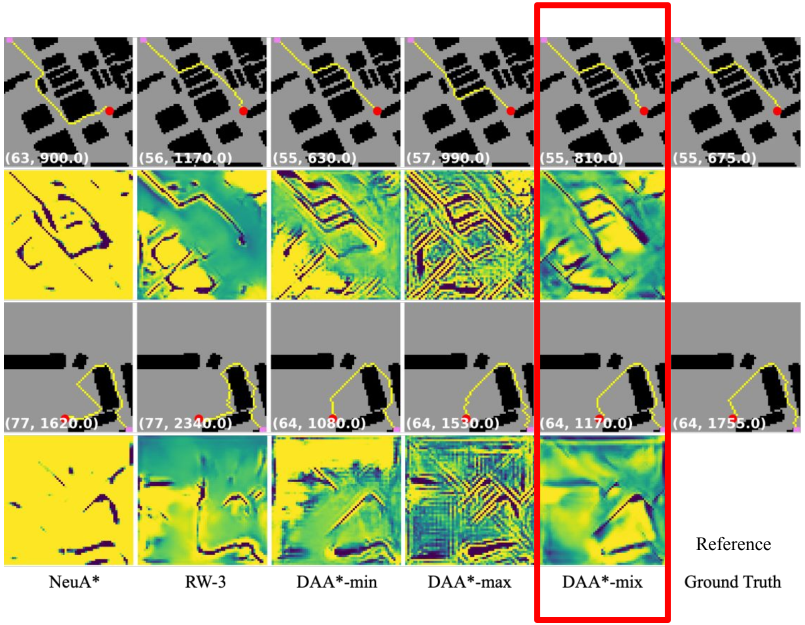
Method	SPR (%)↑	PSIM (%)↑	ASIM (%)↑	Ep (%)↑
		MPD		
A*	**98.70**±**0.0**	35.20±0.0	47.27±0.0	N/A
Theta*	**98.70**±**0.0**	38.00±0.0	53.84±0.0	N/A
Neural A*	91.19±0.4	44.26±0.2	54.93±0.1	44.10±0.3
Rand-walk	82.06±1.1	46.63±0.2	55.72±0.1	0.18±0.1
DAA*-min	91.56±0.1	45.22±0.3	55.53±0.1	53.43±0.4
DAA*-max	93.53±0.3	**47.95**±**0.2**	**58.89**±**0.1**	**70.96**±**0.4**
DAA*-mix	95.56±0.4	47.83±0.1	58.72±0.1	69.99±0.1
		TMPD		
A*	**94.70**±**0.0**	29.40±0.0	54.07±0.0	N/A
Theta*	91.80±0.0	28.60±0.0	51.42±0.0	N/A
Neural A*	78.63±0.6	39.04±0.1	57.47±0.1	49.86±1.1
Rand-walk	78.30±1.1	43.08±0.6	61.16±0.1	9.51±0.4
DAA*-min	81.91±1.4	40.37±0.3	59.03±0.2	63.63±0.4
DAA*-max	80.13±0.4	40.42±0.3	58.93±0.1	**84.03**±**0.7**
DAA*-mix	88.59±0.3	**43.86**±**0.4**	**63.29**±**0.1**	78.90±1.4

Method	SPR (%)↑	PSIM (%)↑	ASIM (%)↑	Ep (%)↑
		CSM		
A*	**94.60**±**0.0**	27.20±0.0	53.61±0.0	N/A
Theta*	93.67±0.0	27.30±0.0	51.53±0.0	N/A
Neural A*	73.83±0.1	38.64±0.2	56.85±0.1	30.92±0.5
Rand-walk	70.38±1.1	43.77±0.2	62.67±0.1	2.57±0.1
DAA*-min	80.16±1.3	42.96±0.7	62.14±0.1	44.70±1.9
DAA*-max	74.08±1.5	40.73±0.6	59.28±0.1	**83.33**±**1.2**
DAA*-mix	82.03±1.1	**44.51**±**0.8**	**64.15**±**0.1**	76.86±1.3

Experiment

Evaluation on binary maps

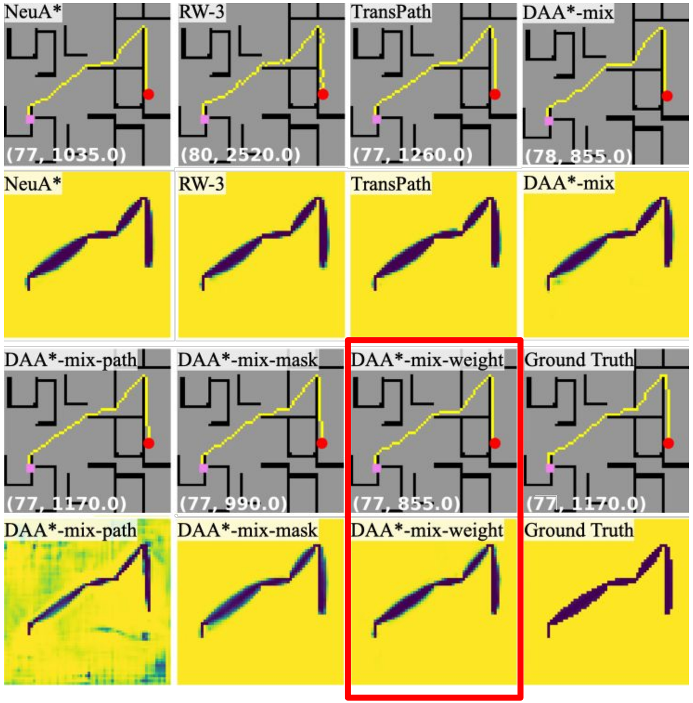




Experiment

Evaluation on binary maps - compare with SOTA TransPath

Method	SPR↑	PSIM↑	ASIM↑	Hist↓
A*	99.08	52.61	52.96	14.59
Theta*	**99.65**	51.76	65.18	10.53
Neural A*	90.92	50.61	62.11	**1.59**
Rand-walk	37.22	45.46	54.31	6.57
TransPath[2]	90.62	49.78	62.53	1.83
DAA*	87.04	53.38	62.14	1.68
DAA*-path	94.23	**56.37**	65.44	4.02
DAA*-mask	96.04	54.91	65.87	4.02
DAA*-weight	96.87	55.73	**66.20**	3.66



Experiment

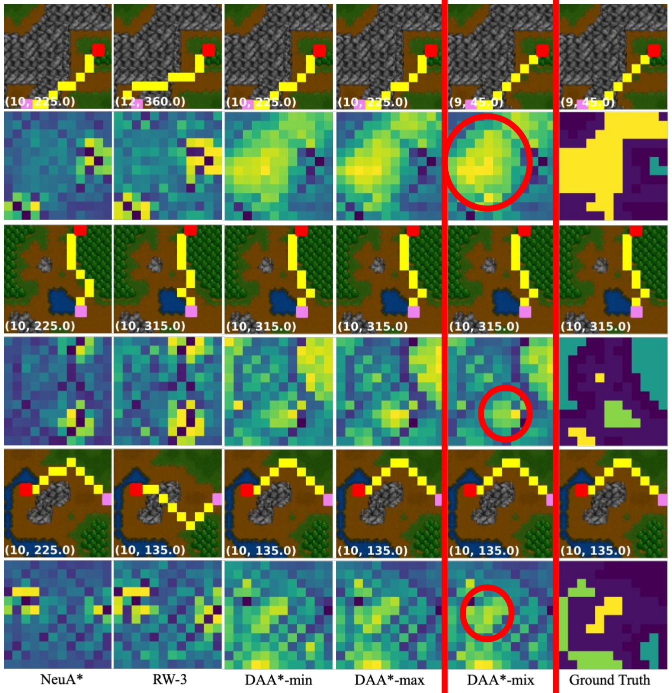
Evaluation on video-game maps

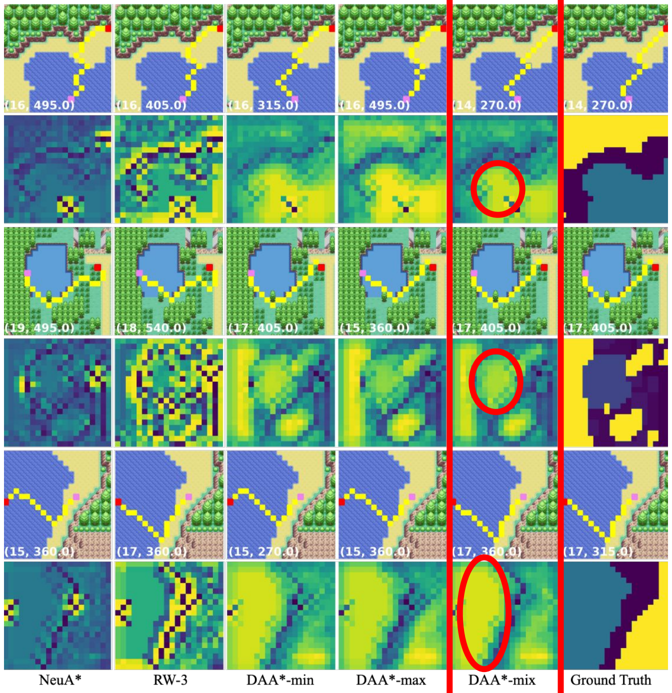
Method	SPR (%)↑	PSIM (%)↑	ASIM (%)↑	CD↓
		Warcraft		
Neural A*	81.03±0.6	70.65±0.6	51.05±0.3	2.85±0.2
Rand-walk	72.68±1.0	67.95±0.4	45.09±0.1	2.94±0.1
DAA*-min	86.55±1.3	**76.71±1.0**	**62.68±0.4**	**2.29±0.2**
DAA*-max	86.83±0.5	75.27±0.4	60.16±0.1	2.47±0.1
DAA*-mix	**89.30±0.5**	75.17±0.4	60.35±0.1	2.53±0.1
		Pokémon		
Neural A*	67.53±2.8	68.84±1.0	55.07±0.3	4.94±0.4
Rand-walk	52.98±1.6	63.60±1.2	43.77±0.1	5.19±0.3
DAA*-min	81.25±3.0	**72.45±0.8**	**61.34±0.2**	**4.81±0.2**
DAA*-max	78.00±2.1	71.11±0.3	58.45±0.1	4.82±0.1
DAA*-mix	**84.70±0.8**	71.49±0.4	60.02±0.1	5.18±0.2

Experiment

Evaluation on video-game maps

learned effective probability map





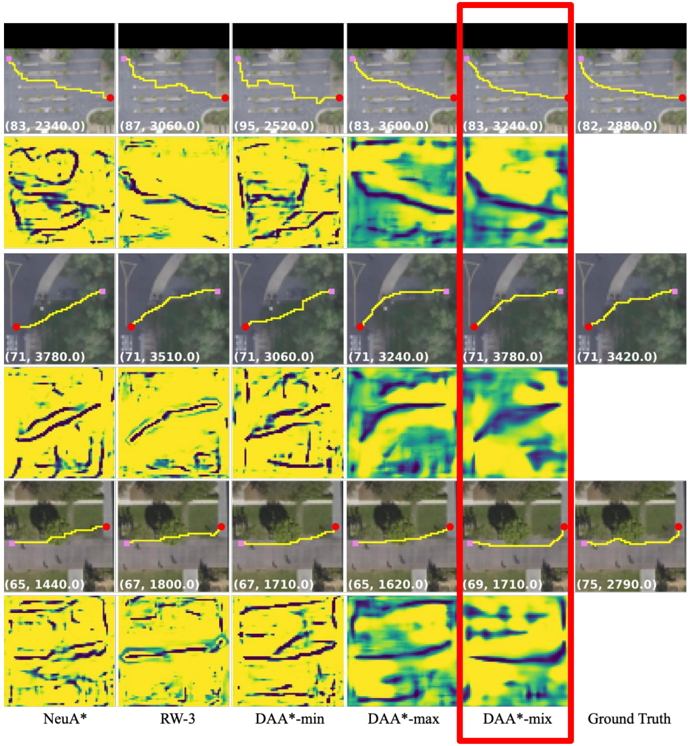
Experiment

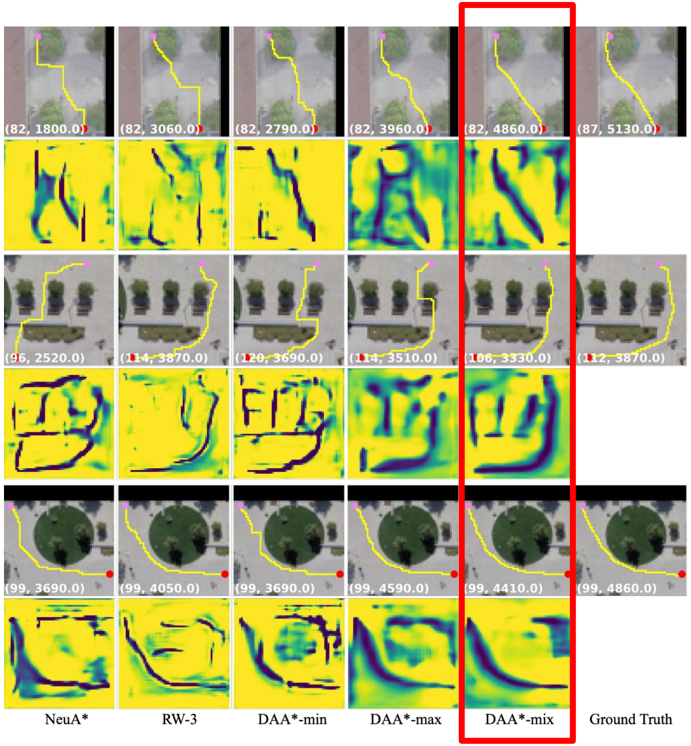
Evaluation on drone-view maps

Method	PSIM (%)↑	ASIM (%)↑	CD↓
	SDD-intra		
Neural A*	40.12±0.4	43.78±0.1	12.25±0.6
Rand-walk	40.22±0.6	43.06±0.1	10.42±0.7
DAA*-min	40.44±0.6	44.37±0.1	11.70±0.6
DAA*-max	41.90±0.7	47.45±0.1	9.39±1.2
DAA*-mix	**42.12±0.4**	**47.82±0.1**	**9.03±0.2**
	SDD-inter		
Neural A*	35.52±0.1	43.52±0.1	23.42±0.9
Rand-walk	36.67±0.3	44.96±0.1	19.86±1.6
DAA*-min	35.71±0.3	44.50±0.1	22.47±0.9
DAA*-max	38.65±0.2	49.51±0.1	**18.92±1.2**
DAA*-mix	**38.78±0.2**	**49.77±0.1**	19.67±0.9

Experiment

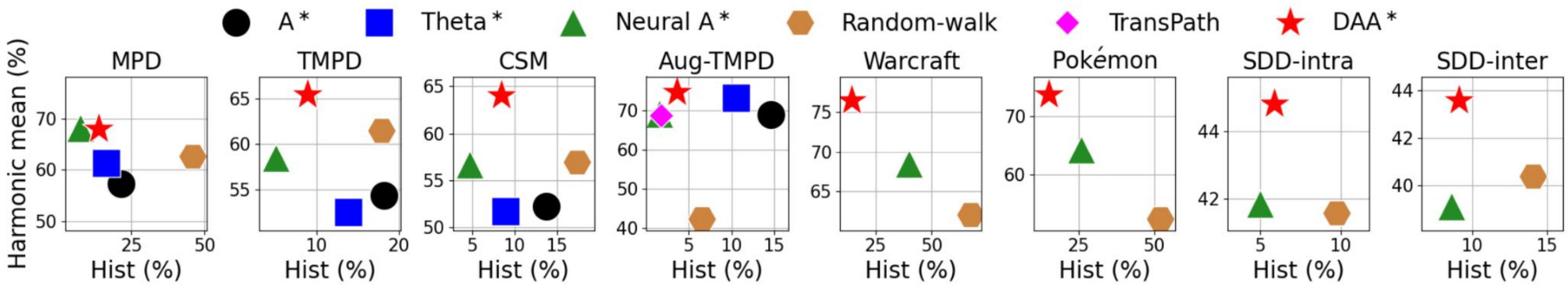
Evaluation on drone-view maps





Experiment

Trade-off between optimality and efficiency



best path optimality (shortest length and high similarity) with considerable cost

Conclusion

- Consider the flexibility of adapting both **path shortening and smoothing**
- End-to-end with **more accessible path labelling** instead of probability map
- **Full analysis and evaluation** of smoothness effects on imitation learning