

# Robust Unfolding Network for HDR Imaging with Modulo Cameras

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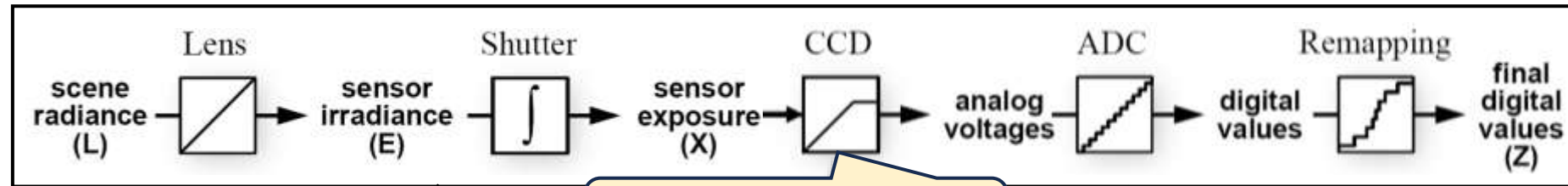


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# High Dynamic Range (HDR) Imaging



Scene with high dynamic range



Low dynamic range (LDR) Image

Due to the physical property of standard camera, natural scene with high dynamic range (contrast) will be captured as the over(under)-exposure areas in the captured photo.

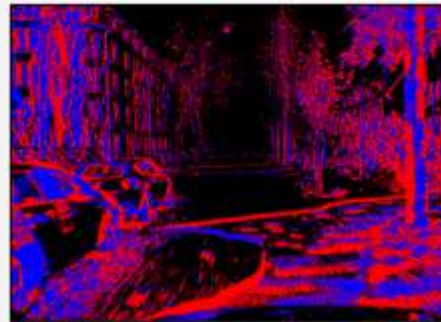
# High Dynamic Range (HDR) Imaging



Multi-exposure HDR



Single-exposure HDR



Event data



Infrared radiation

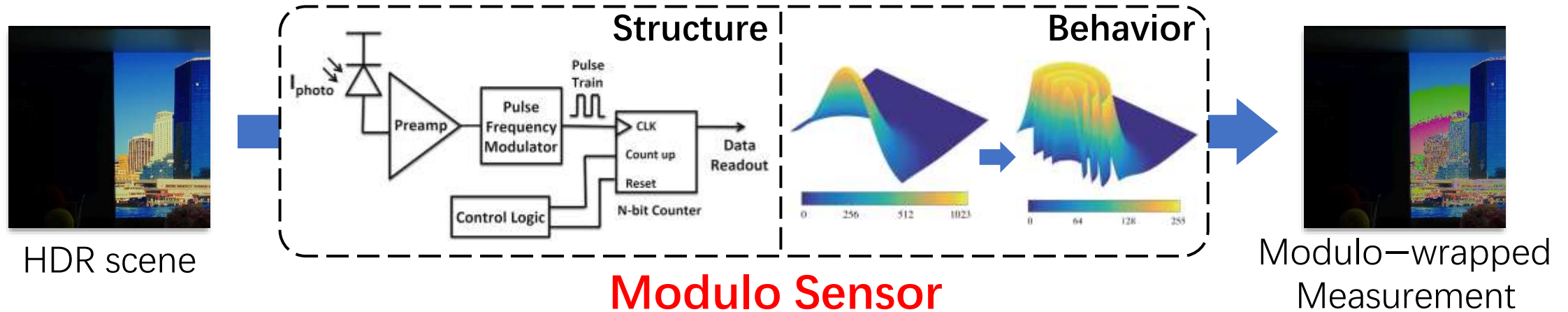


Modulo sample

Captured by  
**modulo camera**

Specific sensing techniques for HDR

# Modulo Camera for HDR Imaging



**Mechanism:** Once the accumulated radiance value reaches pre-defined threshold, *e.g.*, 256 for an 8-bit sensor, it resets to zero and resumes counting, enabling a theoretically unbounded dynamic range.

**Formulation:** Relationship between the HDR image and modulo measurement is

$$\mathbf{X} = \mathcal{M}(\mathbf{Y}), \quad \mathcal{M}: \mathcal{M}(y) = y \bmod a, \quad a = 2^b,$$

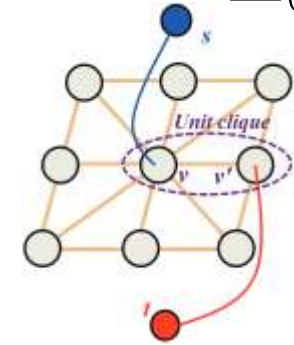
•  $\mathbf{X} \in [0, a)^{H \times W \times C}$ : Modulo-wrapped measurement;  $\mathbf{Y} \in [0, 2^B)^{H \times W \times C}$ : HDR image.

**Goal: Reconstruct the HDR Image  $\mathbf{Y}$  from its modulo-wrapped measurement  $\mathbf{X}$ .**

# Related Works

- Optimization using handcrafted prior.  
*E.g.*, MRF [1] uses graph-cut optimization with smoothness prior enforced.
  - ※ Limited performance on real-world images with complex patterns.
- Optimization using trained deep denoiser.  
*E.g.*, PnP-UA [2] solves a variational regularization model using a pre-trained denoising network.
  - ※ Effectiveness is constrained due to the domain shift.

$$C(k|I_m) = \sum_{(i,j) \in \mathbb{G}} V(|\hat{I}_i - \hat{I}_j|)$$



Seek 2D  
binary sets  
 $\delta \in \{0,1\}$   
that make  
 $C(k + \delta|I_m)$   
 $< C(k|I_m)$ .



Iterative Optimization

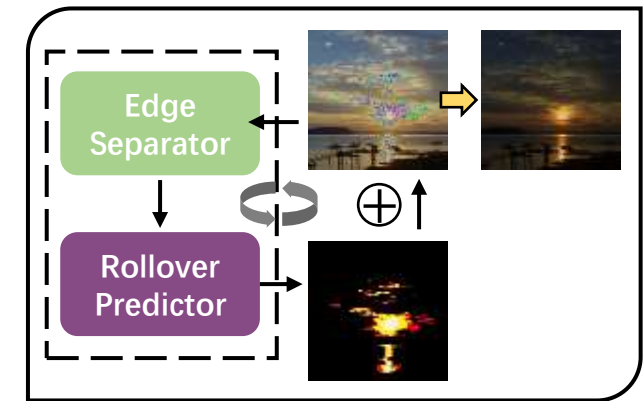
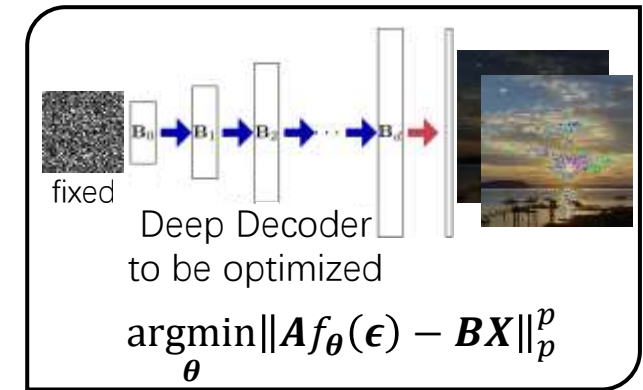
$$\underset{\mathbf{Q}}{\operatorname{argmin}} \|\mathbf{A}\mathbf{Q} - \mathbf{B}\mathbf{X}\|_p^p + \lambda \mathcal{R}_\theta(\mathbf{Q})$$

[1] Zhao Hang, Shi Boxin, Fernandez-Cull Christy, et al. Unbounded High Dynamic Range Photography Using a Modulo Camera. ICCP, 2015.

[2] Bacca Jorge, Monroy Brayan, Arguello Henry. Deep Plug-and-Play Algorithm for Unsaturated Imaging. ICASSP, 2024.

# Related Works

- Optimization leveraging untrained NN prior.  
*E.g.*, [3] optimizes a TV-regularized variational model using an untrained decoder, leveraging deep image prior.
  - ✱ **High computational cost and inferior performance.**
- Supervised End-to-end Network.  
*E.g.*, UnModNet [4] formulates modulo inversion as a rollover classification problem.
  - ✱ **Insufficient generalization as it disregards the inherent properties of the physical imaging process.**





# Contributions

- Reformulation of the HDR reconstruction with modulo cameras in the gradient space, leveraging Itoh's continuity condition.
- The 1<sup>st</sup> deep unfolding network for HDR reconstruction with modulo cameras, derived from a variational model using the reformulation, with an auxiliary variable to effectively handle outliers.
- A spiking neuron-based module to learn a sparsity-related prior for outliers, enhancing the performance of HDR reconstruction.
- Superior performance over existing methods on synthetic & real datasets.

# Reformulation of Modulo HDR

**Proposition 1.** *[Extension of 2D Itoh's continuity condition [5]]*

Given an HDR image  $\mathbf{Y}$  and its modulo counterpart  $\mathbf{X} = \mathcal{M}(\mathbf{Y})$ . Let  $\tilde{\mathcal{M}}$  denote another modulo operator defined by  $\tilde{\mathcal{M}}(y) = [(y + a/2) \bmod a] - a/2$ . For any point with index  $(i, j)$  satisfying  $\|\nabla \mathbf{Y}_{i,j}\|_{\infty} = \max\{|\nabla_x \mathbf{Y}_{i,j}|, |\nabla_y \mathbf{Y}_{i,j}|\} < a/2$ , we have

$$\tilde{\mathcal{M}}(\nabla \mathbf{X}_{i,j}) = \nabla \mathbf{Y}_{i,j}.$$

This property simplifies the HDR reconstruction by allowing direct inversion of the modulo operation in the gradient domain for pixels within this bound.



# Optimization Model

- Introducing an auxiliary variable  $\mathbf{V}$  to mitigate the impact of outliers, define an optimization problem for modulo HDR reconstruction:

$$\min_{\mathbf{Y}, \mathbf{V}} \|\nabla \mathbf{Y} - (\tilde{\mathcal{M}}(\nabla \mathbf{X}) - \mathbf{V})\|_{\text{F}}^2 + \Phi(\mathbf{Y}) + \Theta(\mathbf{V}).$$

sparsity  
regularization

- Unfold its iterative scheme of sub-problems using proximal gradient descent solver.

In  $k^{\text{th}}$  iteration,

$$\mathbf{Q}_k^{(0)} = \mathbf{P}_k^{(0)} = \mathbf{Y}_{k-1}, \beta^{(0)} = 1.$$

For  $n = 1$  to  $N$ ,

$$\begin{cases} \mathbf{Q}_k^{(n)} = \mathbf{P}_k^{(n-1)} + \eta_k \mathcal{G}_{\mathbf{P}}[\nabla \mathbf{P}_k^{(n-1)} - (\tilde{\mathcal{M}}(\nabla \mathbf{X}) - \mathbf{V}_{k-1})], \\ \beta^{(n)} = (1 + \sqrt{1 + 4\beta^{(n-1)} \cdot \beta^{(n-1)}})/2, \\ \mathbf{P}_k^{(n)} = \mathbf{Q}_k^{(n)} + \frac{\beta^{(n-1)} - 1}{\beta^{(n)}} \cdot (\mathbf{Q}_k^{(n)} - \mathbf{Q}_k^{(n-1)}), \end{cases}$$

#Accelerated gradient descent for better convergence.

$$\mathbf{Y}_k = \text{UNet}_{\mathbf{Y}}(\mathbf{Q}_k^{(N)}, \mathbf{X}), \quad (1)$$

$$\mathbf{E}_k = \tilde{\mathcal{M}}(\nabla \mathbf{X}) - \nabla \mathbf{Y}_k,$$

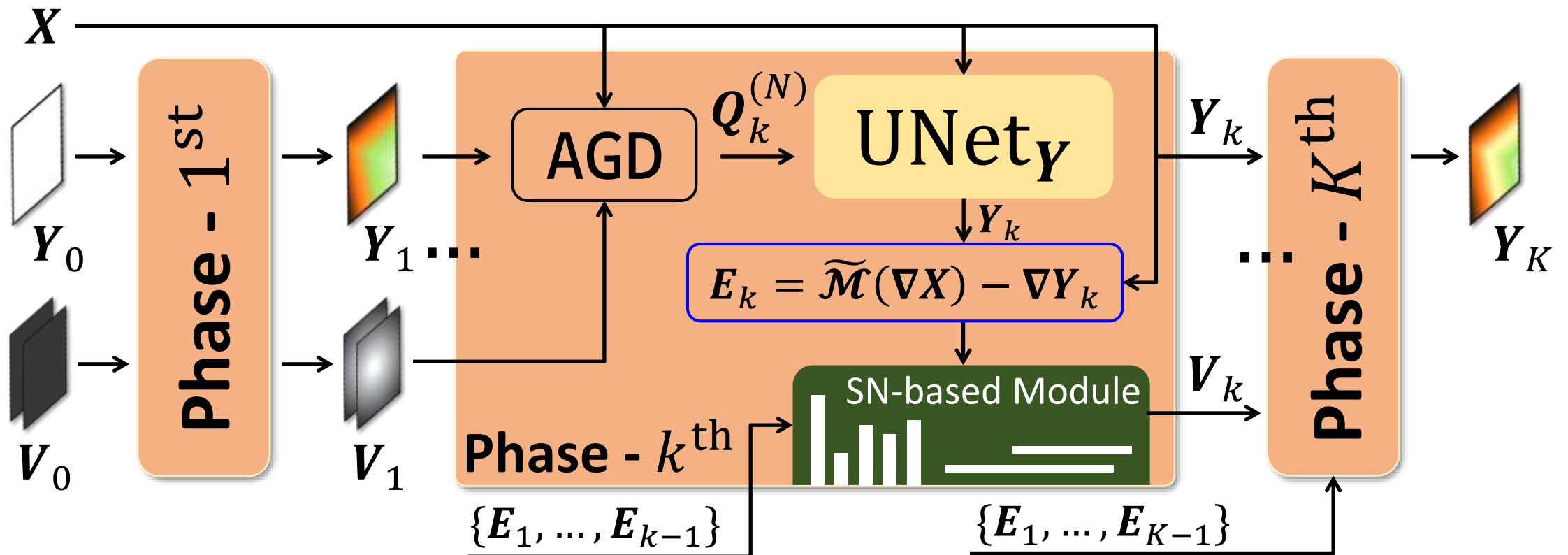
$$\mathbf{V}_k = \text{SNM}(\mathbf{E}_1, \dots, \mathbf{E}_k); \quad (2)$$

#(1) Replace the step for  $\Phi(\mathbf{Y})$  by UNet.

#(2) Replace the step for  $\Theta(\mathbf{V})$ : Learn a spiking neuron-based module (SNM) for capturing the structures of sparse outliers.

# Deep Unfolding Network

- Implement each iteration as a phase block in the proposed DUN:



# Spiking Neuron-based Module

- Employ the Leaky Integrate-and-Fire (LIF) model [6].

The spiking neuron aggregates outlier information across unfolding phases, enhancing the model's ability to adaptively suppress sparse erroneous measurements and improve reconstruction accuracy.

## $l^{\text{th}}$ Spiking Neuron

Initialize  $\mathbf{U}_l^{(0)} = \mathbf{O}_l^{(0)} = \mathbf{0}$ .

For  $t = 1, \dots, 2k$ ,

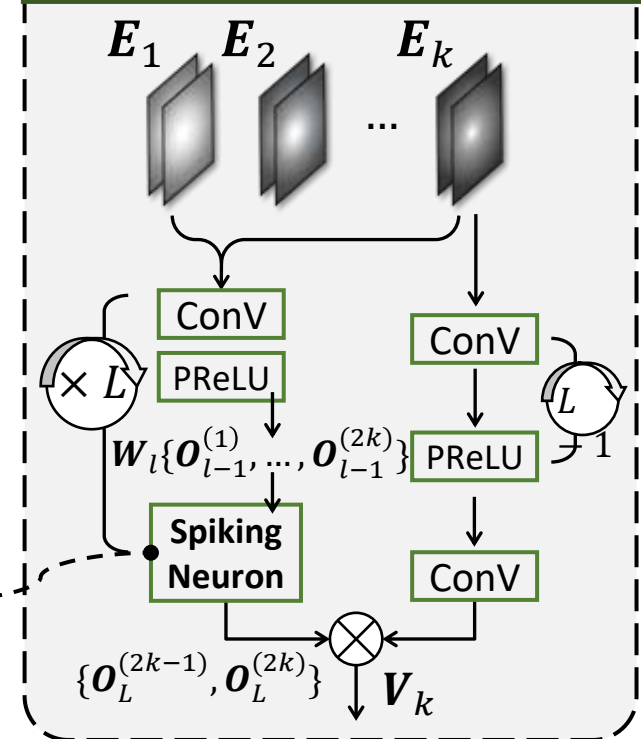
$$\mathbf{U}_l^{(t)} = \kappa \mathbf{U}_l^{(t-1)} (1 - \mathbf{O}_l^{(t-1)}) + \mathbf{W}_l \cdot \mathbf{O}_{l-1}^{(t)},$$

$$\mathbf{O}_l^{(t)} = \mathcal{T}(\mathbf{U}_l^{(t)}, \tau).$$

#Thresholding function for spiking.

Output  $\{\mathbf{O}_l^{(1)}, \dots, \mathbf{O}_l^{(2k)}\}$ .

## SN-based Module



# Training Strategy

- The outputs of all phases in the DUN ( $k = 1, \dots, K$ ) are supervised using the overall loss function:

$$\mathcal{L}_{\text{all}} := \sum_{k=1}^K \gamma_k \left( \mathcal{L}_{\text{g}}(\mathbf{Y}_k, \mathbf{Y}) + \rho \mathcal{L}_{\text{s}}(\mathbf{Y}_k, \mathbf{Y}) \right),$$

where  $\rho \in \mathbb{R}^+$ ,  $\gamma_k = 1/(K - k + 1)$ , and

$$\mathcal{L}_{\text{g}}(\mathbf{Y}_k, \mathbf{Y}) := \frac{1}{2HWC} \|\nabla \mathbf{Y}_k - \nabla \mathbf{Y}\|_1,$$

$$\mathcal{L}_{\text{s}}(\mathbf{Y}_k, \mathbf{Y}) := \frac{1}{HWC} \sum_{i,j,c} \log(\cosh(\mathbf{Y}_k - \mathbf{Y})_{i,j,c}).$$

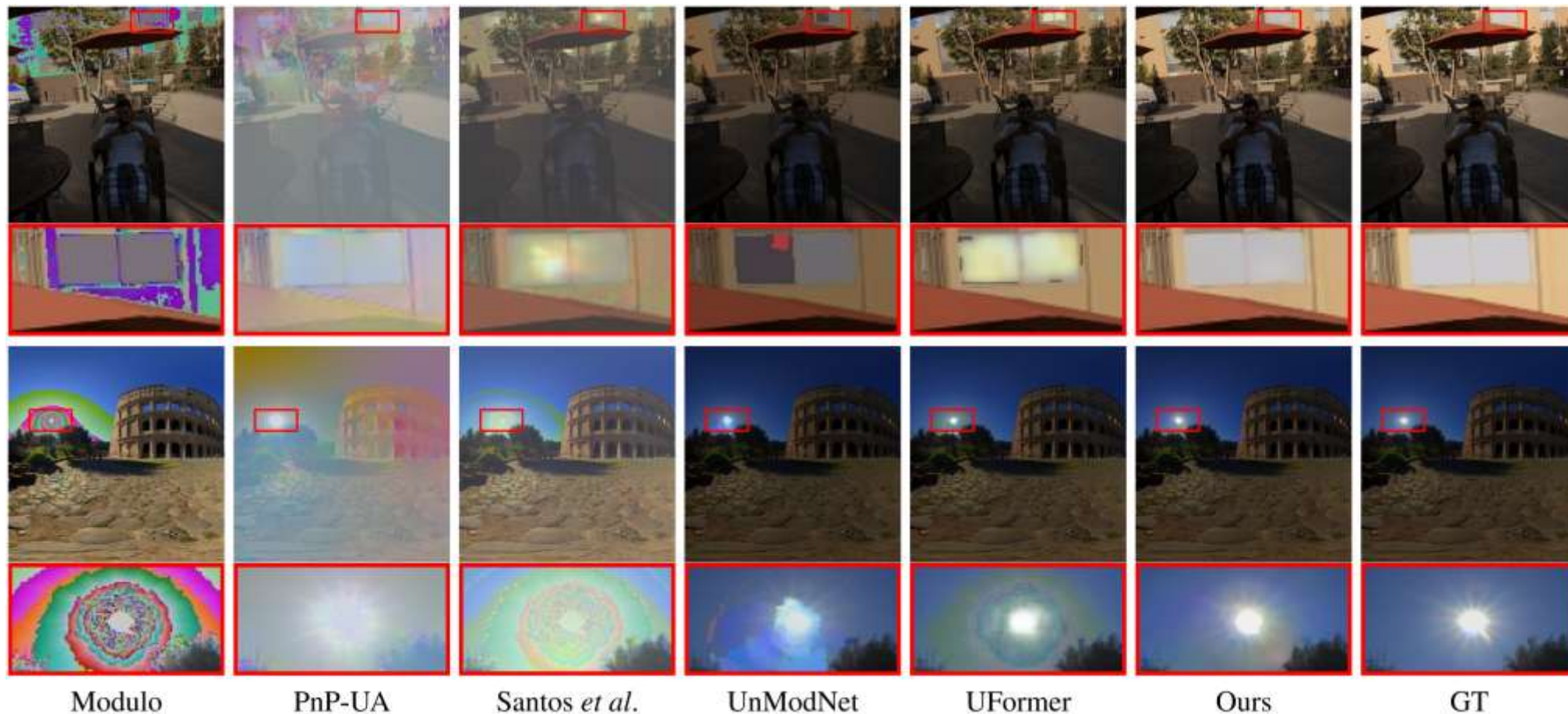
# Evaluation on Synthetic Dataset

**Boldfaced:** best results.

| Method                            | NRMSE(%)    | PSNR(dB)     | SSIM        | MS-SSIM     | Param(M) | FLOPs(G) |
|-----------------------------------|-------------|--------------|-------------|-------------|----------|----------|
| MRF                               | 45.15       | 25.08        | 0.38        | 0.55        | /        | /        |
| PnP-UA                            | 28.35       | 29.38        | 0.34        | 0.51        | 32.64    | 574.44   |
| UnModNet                          | 6.64        | 40.42        | 0.98        | 0.97        | 40.83    | 310.46   |
| ExpandNet <sup>†</sup>            | 11.74       | 21.20        | 0.55        | 0.59        | 0.46     | 53.80    |
| ExpandNet                         | 10.18       | 23.57        | 0.80        | 0.78        | 0.46     | 53.80    |
| Santos <i>et al.</i> <sup>†</sup> | 19.52       | 10.89        | 0.21        | 0.32        | 51.54    | 75.77    |
| Santos <i>et al.</i>              | 7.85        | 39.09        | 0.98        | 0.97        | 51.54    | 75.77    |
| UFormer                           | 7.15        | 42.34        | 0.98        | 0.98        | 20.60    | 164.36   |
| Ours                              | <b>5.56</b> | <b>42.56</b> | <b>0.99</b> | <b>0.98</b> | 4.15     | 62.73    |

Our DUN achieves the highest performance, while maintaining the second-lowest parameter count and FLOPs.

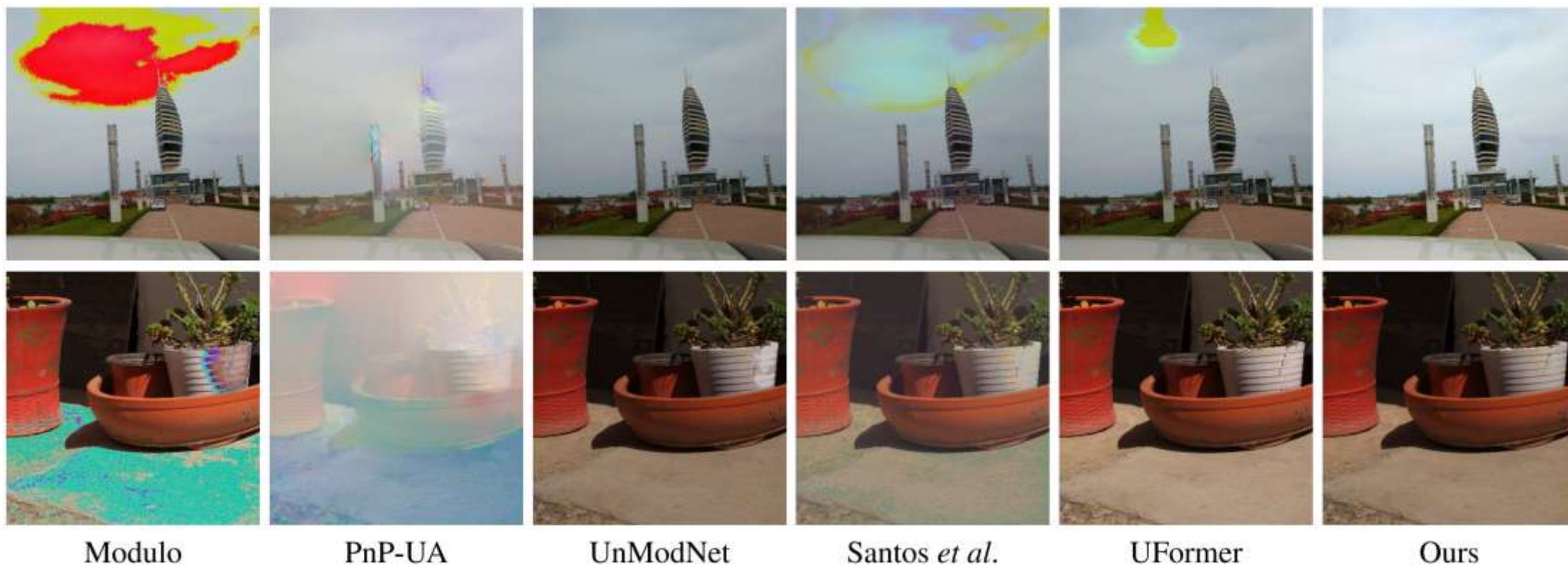
# Visualization on Synthetic Dataset



Our DUN provides the HDR image results with best visual quality.



# Evaluation on Real-RGB Dataset



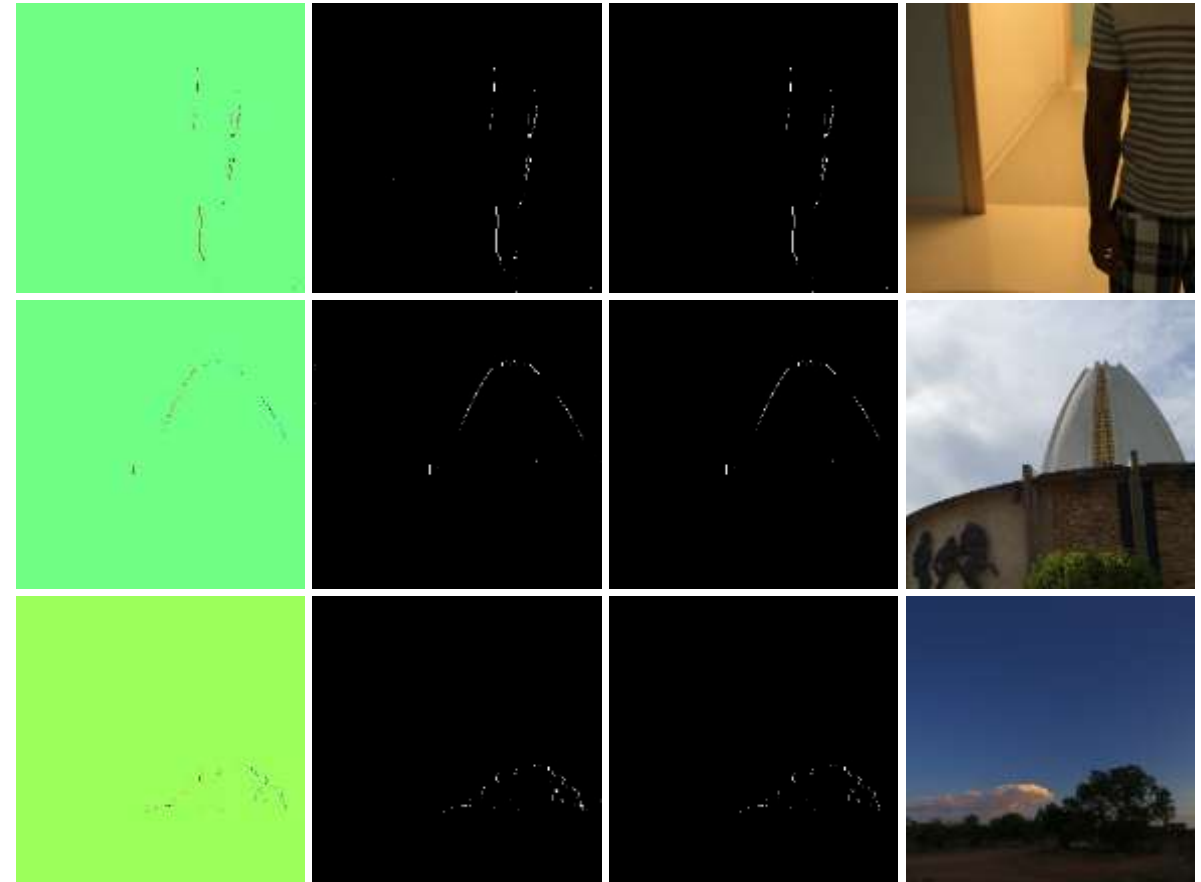
Our DUN yields the most authentic visual results for both samples.

# Ablation Study and Analysis

- Main components

| Baseline                     | NRMSE(%) | PSNR(dB) | SSIM | MS-SSIM |
|------------------------------|----------|----------|------|---------|
| w/o $V$                      | 6.56     | 41.43    | 0.98 | 0.98    |
| SN $\rightarrow$ Th          | 6.30     | 41.79    | 0.98 | 0.98    |
| AGD $\rightarrow$ GD         | 5.97     | 41.55    | 0.98 | 0.98    |
| LogCosh $\rightarrow \ell_1$ | 5.72     | 42.28    | 0.99 | 0.98    |
| Original                     | 5.56     | 42.56    | 0.99 | 0.98    |

- Visualization of  $V$



Predicted  $V$

Binary map

GT outliers

GT HDR image

# Conclusion and Future Work

- To conclude

- ✓ Reformulating the HDR reconstruction with modulo cameras, simplifying the reconstruction as direct inversion in gradient space for most pixels.
- ✓ Constructing the first deep unfolding network for modulo HDR, with an auxiliary variable to effectively handle outliers.
- ✓ Integrating spiking neuron-based modules to learn sparsity-related prior.

- In future

- Enhancing the robustness mechanism against outliers.
- Extending the DUN for other inverse problems involving modulo operators.

## Take home messages

- HDR reconstruction with modulo cameras can be simplified by utilizing the modulo-wrapped gradients of measurement.
- Incorporating physics into NN yields better performance and reduced complexity.
- A spiking neuron-based module is capable of enforcing sparsity regularization with cumulative memory characteristics.

Thank you!