Engineering, Computer Science

Stochastic gradient estimation for higher-order differentiable rendering

THURSDAY, 14:45, Poster session 6, Poster 334

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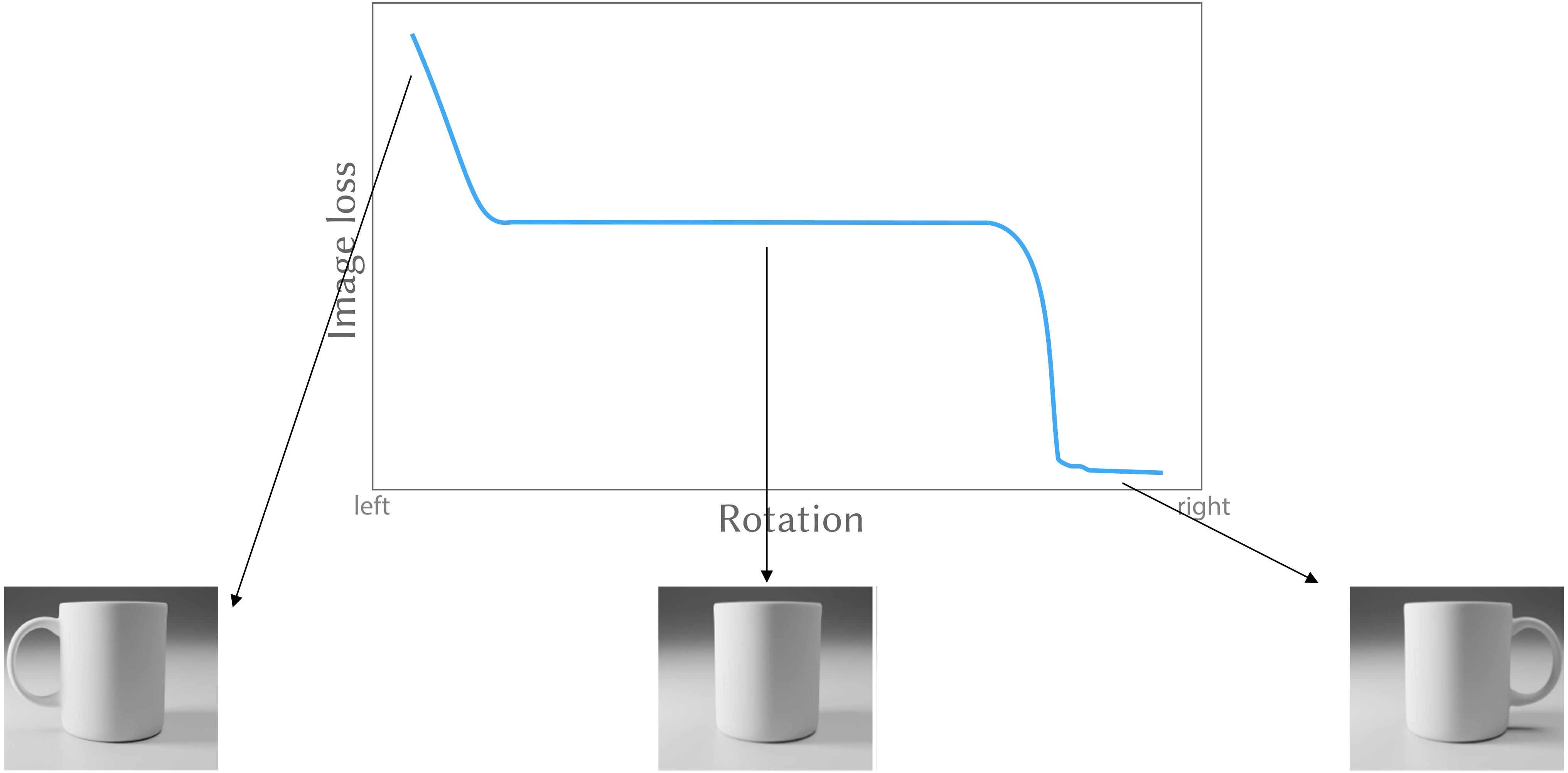
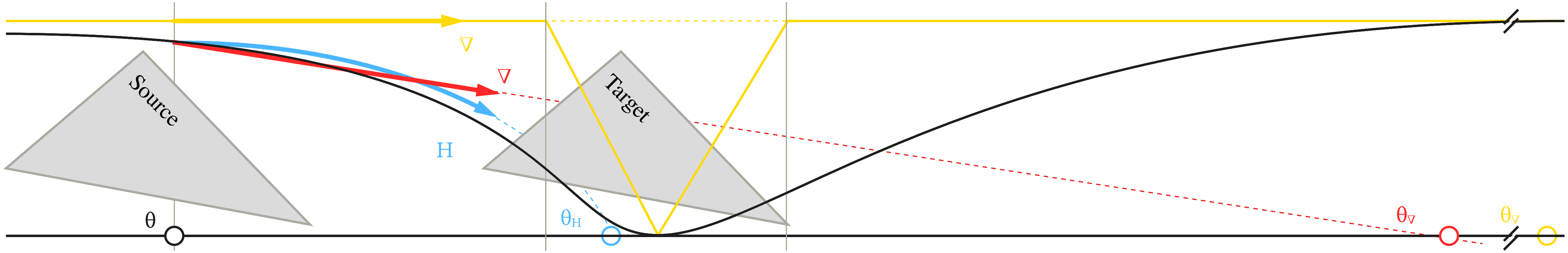
University College London, Adobe Research

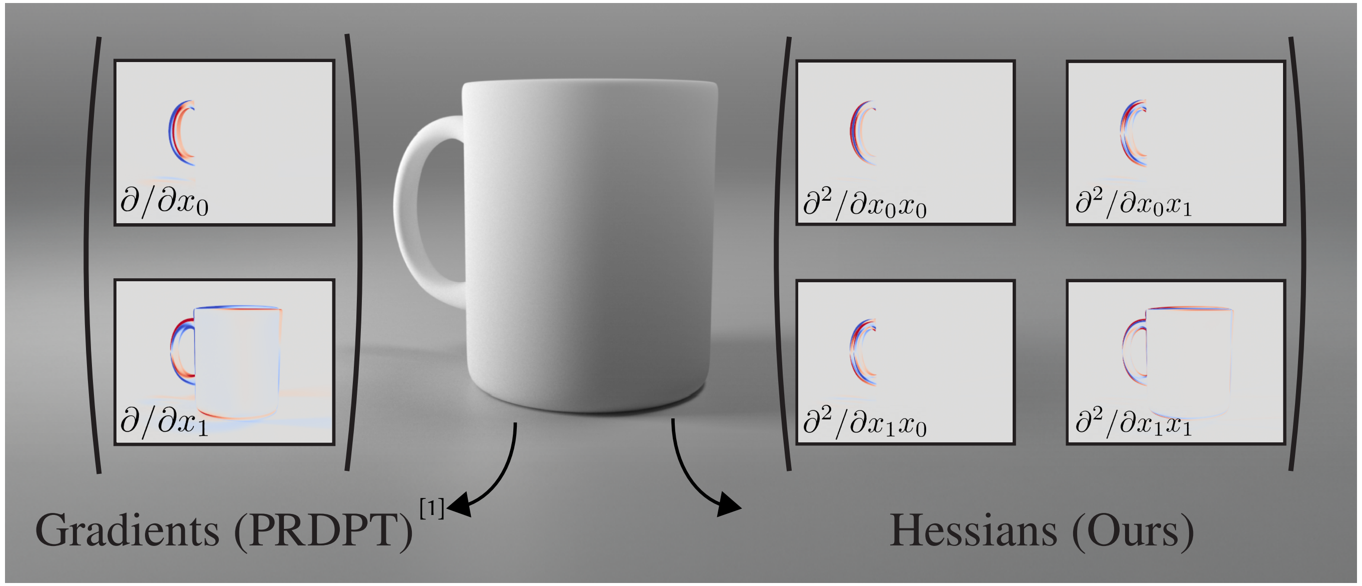
ICCV 2025

Background



Background





[1] Michael Fischer, and Tobias Ritschel. "Plateau-reduced differentiable path tracing." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition.* 2023.

Key idea

$$L(\mathbf{x}, \omega_\mathrm{o}; \boldsymbol{\theta}) = \int_\Omega \underbrace{f_\mathrm{r}(\omega_\mathrm{i}, \omega_\mathrm{o}) L(\mathbf{y}, \omega_\mathrm{i}; \boldsymbol{\theta})(\omega_\mathrm{i} \cdot \mathbf{n})}_{R(\omega_\mathrm{i}; \boldsymbol{\theta})} \mathrm{d}\omega_\mathrm{i}$$

$$\kappa * L(\mathbf{x}, \omega_\mathrm{o}; \boldsymbol{\theta}) = \int_\Omega \int_\Theta \kappa(\boldsymbol{\tau}) R(\omega_\mathrm{i}; \boldsymbol{\theta} - \boldsymbol{\tau}) \mathrm{d}\boldsymbol{\tau} \mathrm{d}\omega_\mathrm{i}$$

$$\mathrm{D}\, \kappa * L(\mathbf{x}, \omega_\mathrm{o}; \boldsymbol{\theta}) = D \int_\Omega \int_\Theta \kappa(\boldsymbol{\tau}) R(\omega_\mathrm{i}; \boldsymbol{\theta} - \boldsymbol{\tau}) \mathrm{d}\boldsymbol{\tau} \mathrm{d}\omega_\mathrm{i}$$

Key idea

First order

Second order

$$\mathrm{D}\,\kappa * L(\mathbf{x}, \omega_\mathrm{o}; \boldsymbol{\theta}) = \int_\Omega \int_\Theta \mathrm{D}\,\kappa(\boldsymbol{\tau}) R(\omega_\mathrm{i}; \boldsymbol{\theta} - \boldsymbol{\tau}) \mathrm{d}\boldsymbol{\tau} \mathrm{d}\omega_\mathrm{i}$$

Gradient

$\mathrm{D}^{\mathrm{G}}\,\kappa(\boldsymbol{\tau})$

Hessian

$\mathrm{D}^{\mathrm{H}}\,\kappa(\boldsymbol{\tau})$

$\mathbf{H}^{-1} v$

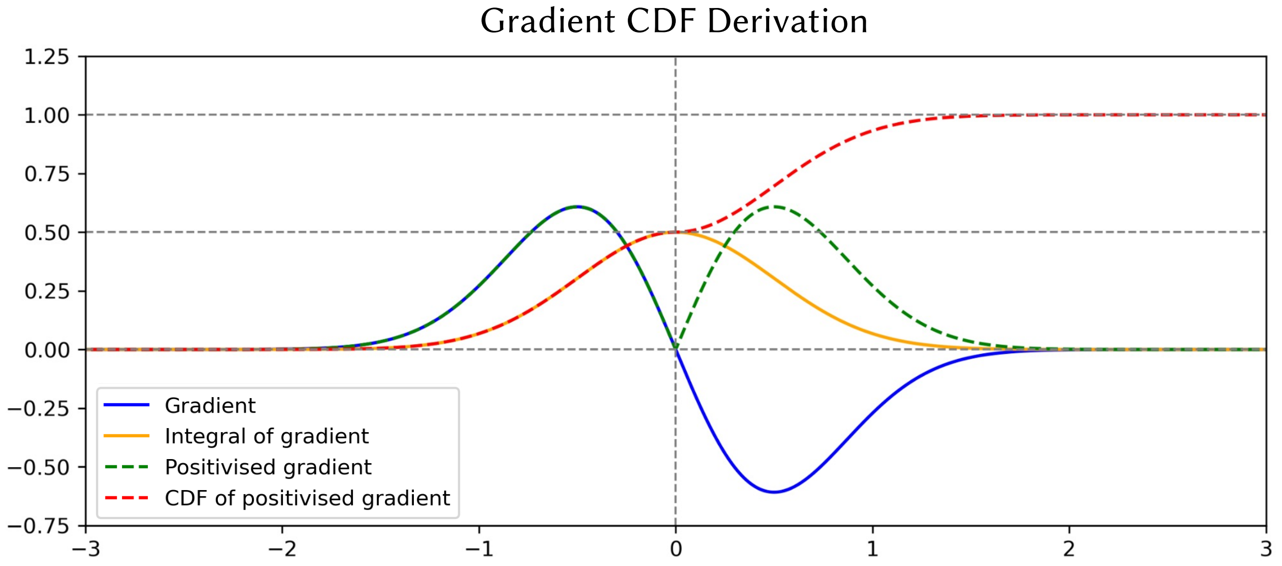
Hessian vector product

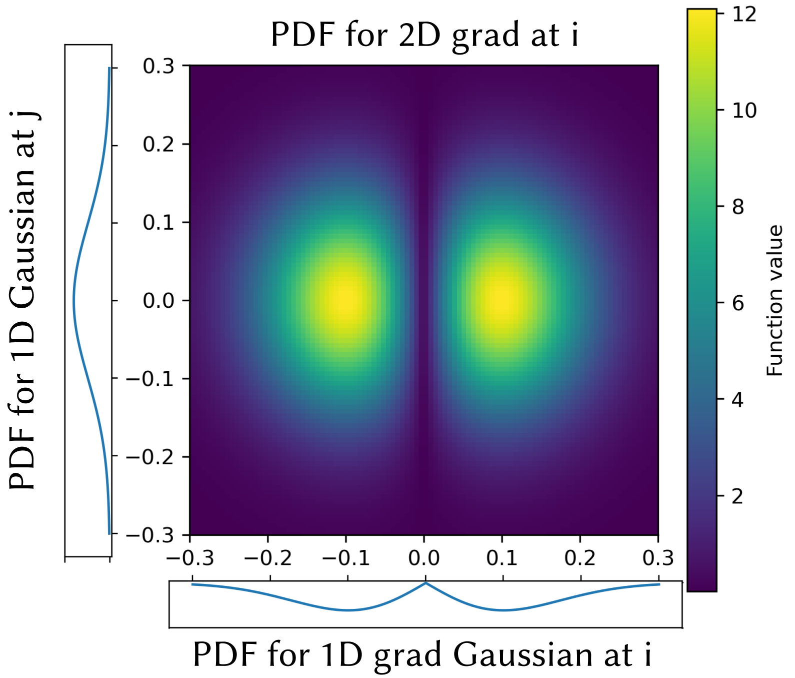
$$\mathrm{D}\,\kappa * L(\mathbf{x}, \omega_\mathrm{o}; \boldsymbol{\theta}) = D \int_\Omega \int_\Theta \kappa(\boldsymbol{\tau}) R(\omega_\mathrm{i}; \boldsymbol{\theta} - \boldsymbol{\tau}) \mathrm{d}\boldsymbol{\tau} \mathrm{d}\omega_\mathrm{i}$$

$\mathrm{D}^{\mathrm{HVP}}\,\kappa(\boldsymbol{\tau})$

Gradient case

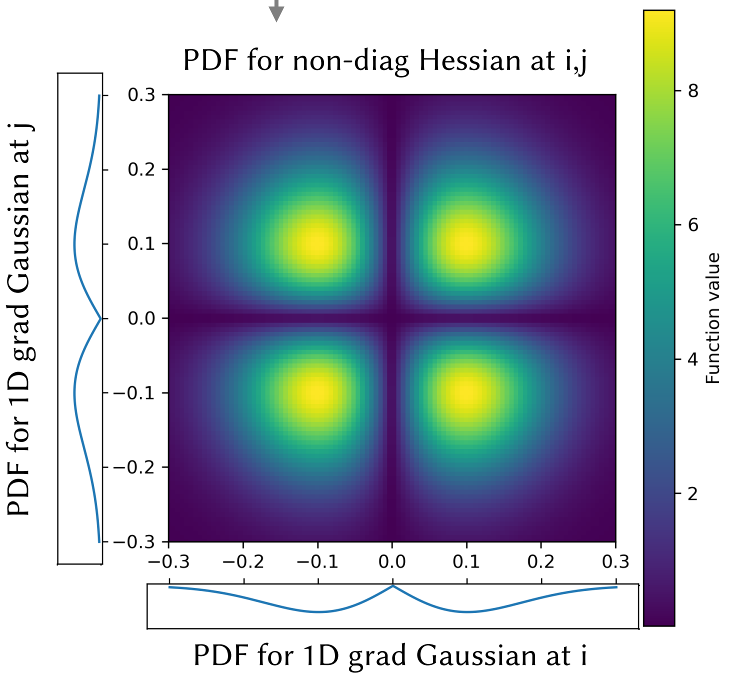
$$\mathrm{D}^{\mathrm{G}}\,\kappa(\boldsymbol{\tau}) = -\frac{\tau_i}{\sigma^2}\cdot\mathcal{N}(\boldsymbol{\tau},\sigma)$$

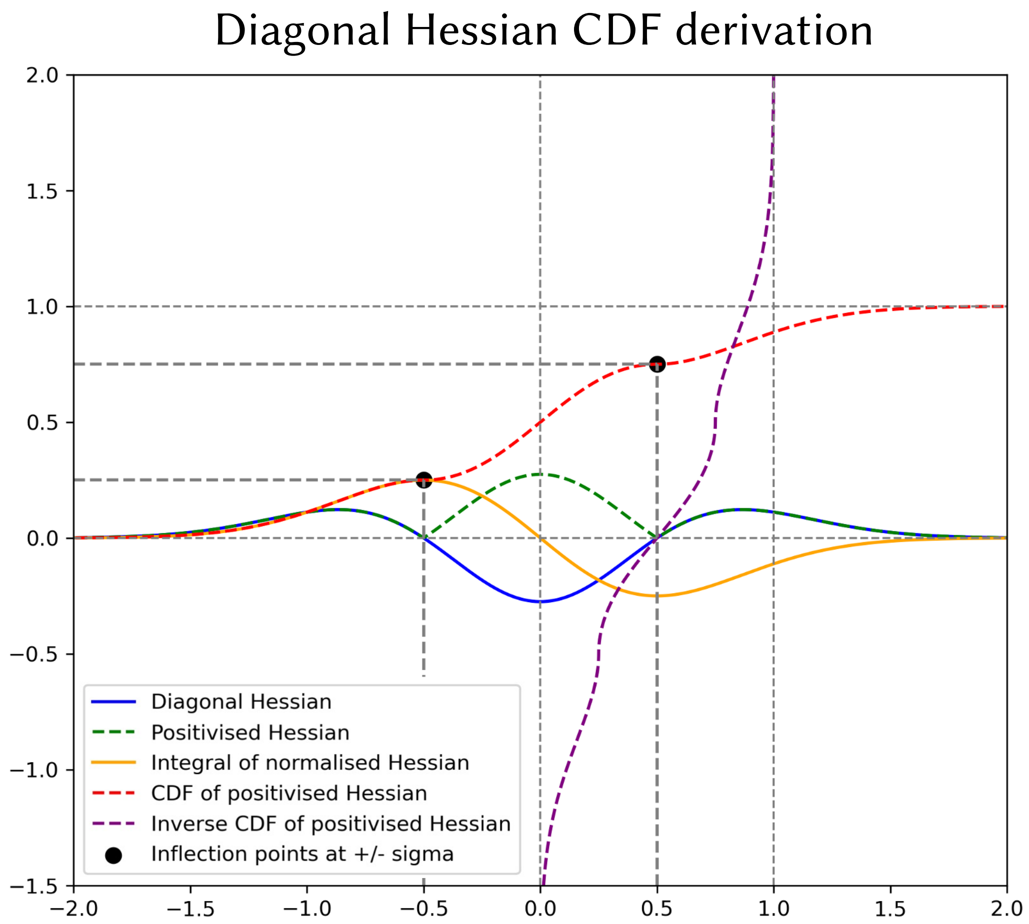




Hessian case

$$\mathrm{D}^{\mathrm{H}}\,\kappa_{i,j}(\boldsymbol{\tau}) = \begin{cases} \left(-\frac{1}{\sigma^2} + \frac{\tau_i^2}{\sigma^4}\right) \cdot \mathcal{N}(\boldsymbol{\tau}, \sigma) & \texttt{if } i = j, \\ \frac{\tau_i \tau_j}{\sigma^4} \cdot \mathcal{N}(\boldsymbol{\tau}, \sigma) & \texttt{else.} \end{cases}$$





HVP case

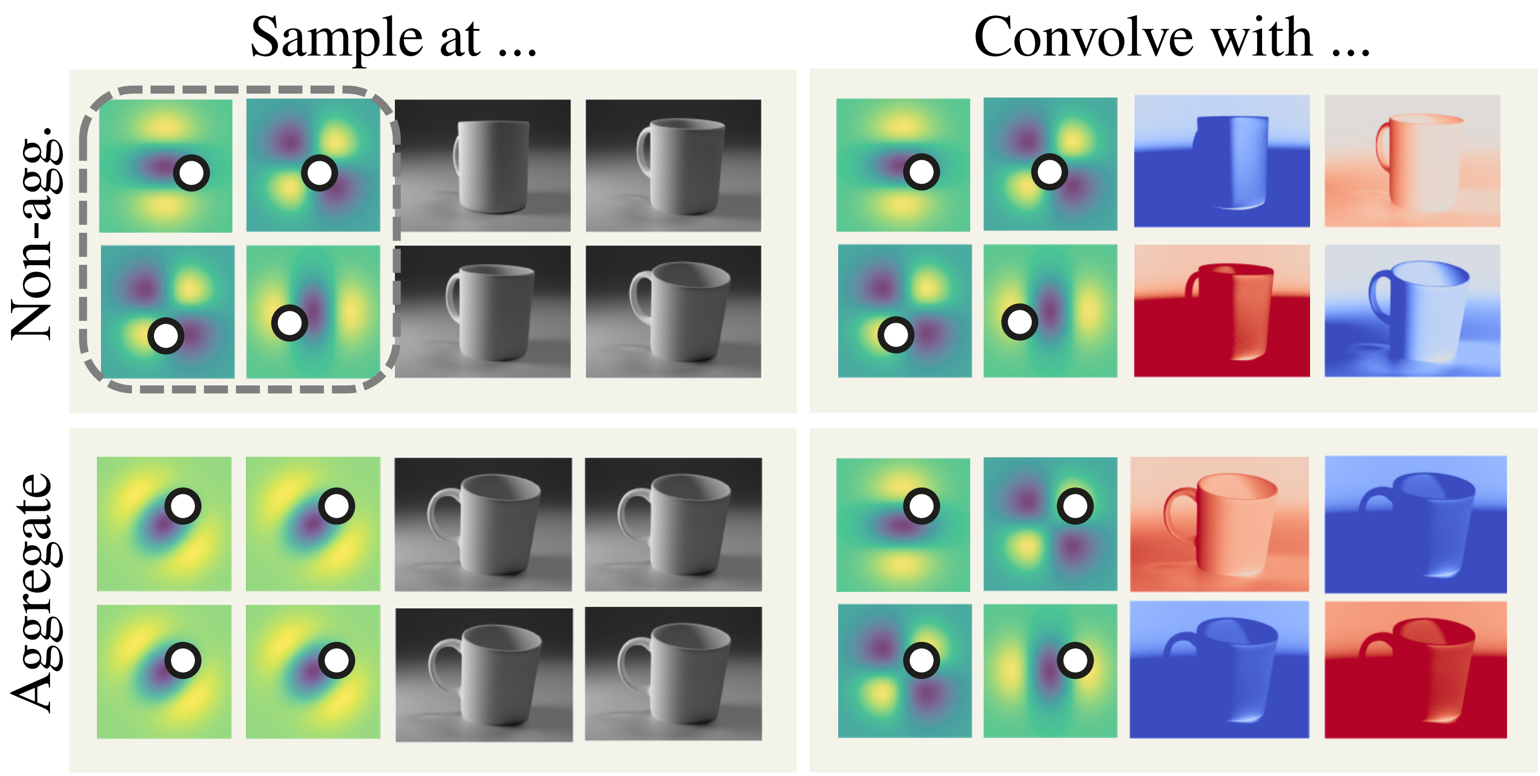
$$\mathrm{D}^{\mathrm{HVP}}\,\kappa(\boldsymbol{\tau}) = \frac{\mathrm{D}^{\mathrm{G}}\,\kappa(\boldsymbol{\tau} - \varepsilon\mathbf{v}) - \mathrm{D}^{\mathrm{G}}\,\kappa(\boldsymbol{\tau} + \varepsilon\mathbf{v})}{2\varepsilon}$$

[2] Pearlmutter, Barak A.. "Fast Exact Multiplication by the Hessian." Neural Computation 6 (1994): 147-160.

Aggregate sampling

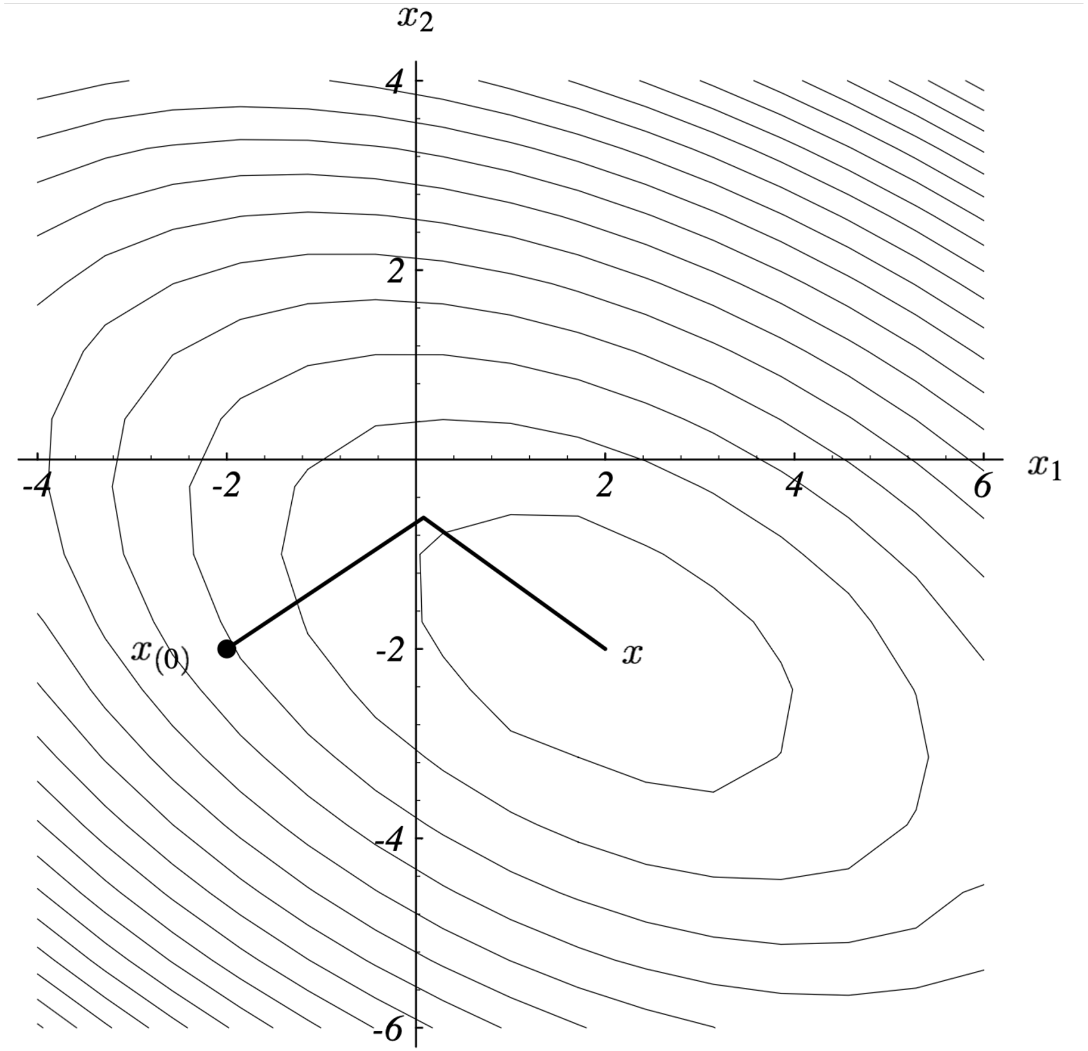
- Need to sample all dimensions for a Hessian $O(n^2)$

- Sample the average of the dimensions

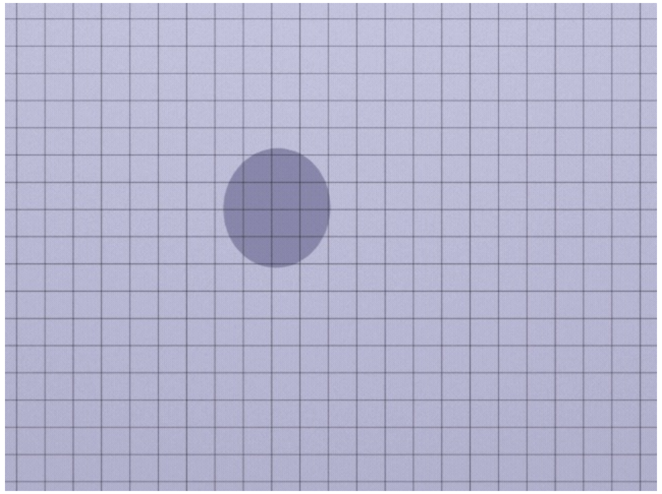


Optimizers

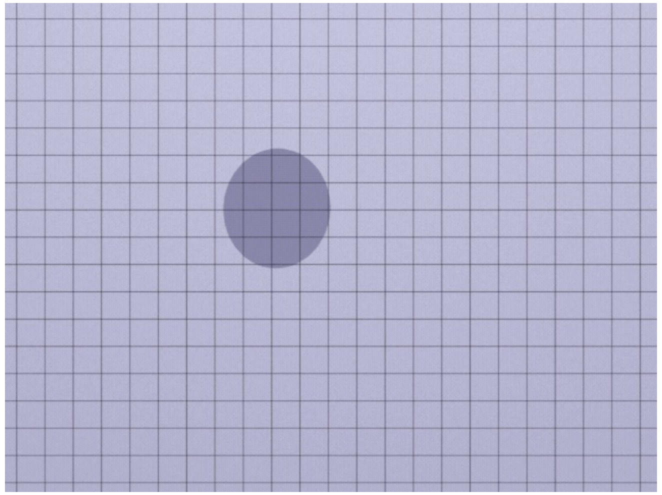
- Conjugate gradients
- Works with Hessian vector products



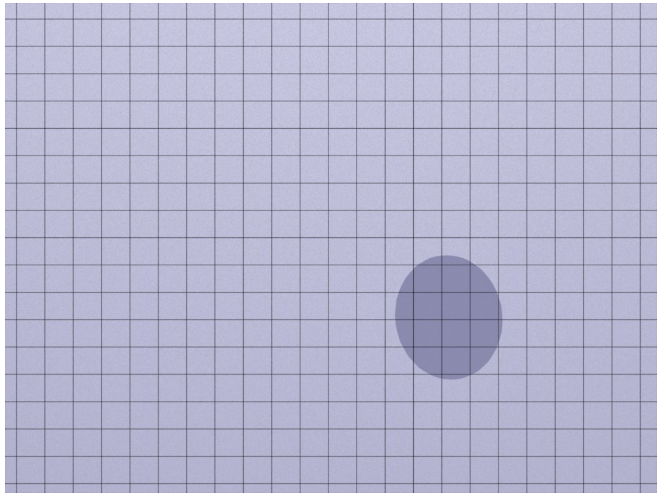
Results



Ours

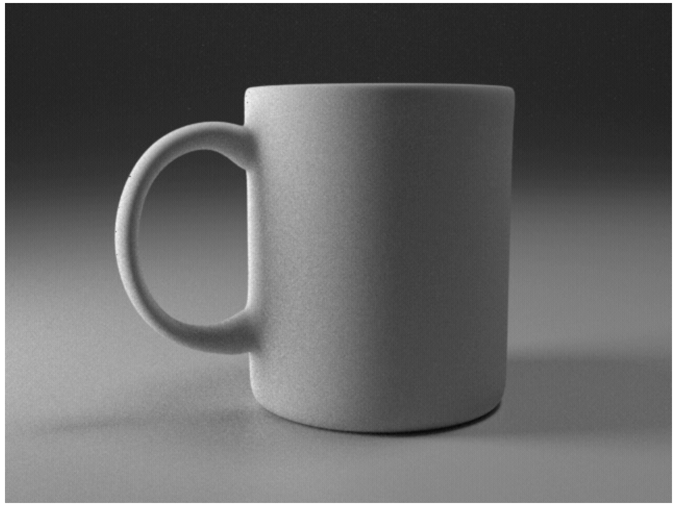


Non-smooth



Ground truth

Results

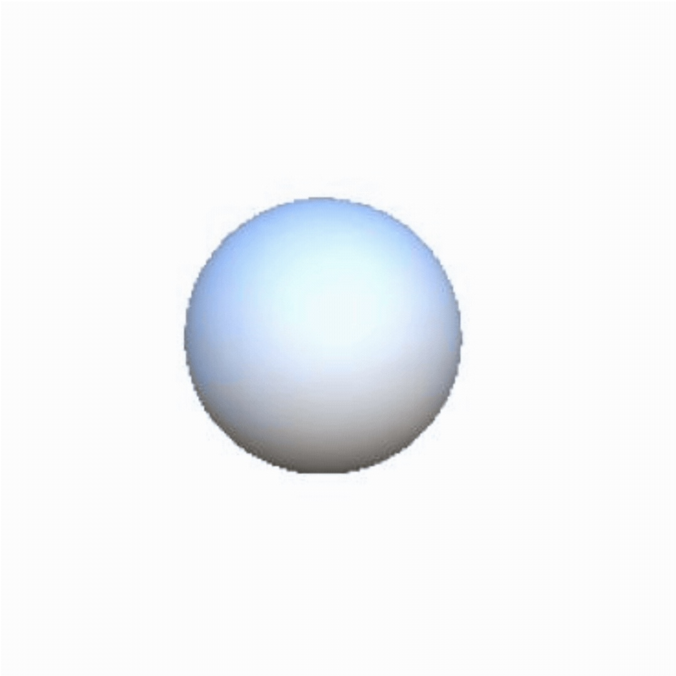


Ours

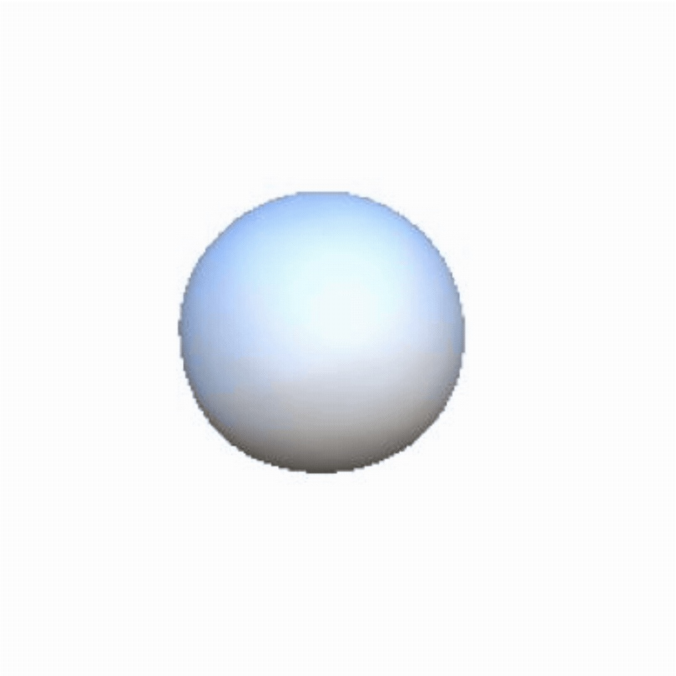
First order

Ground truth

Results



Ours



First order

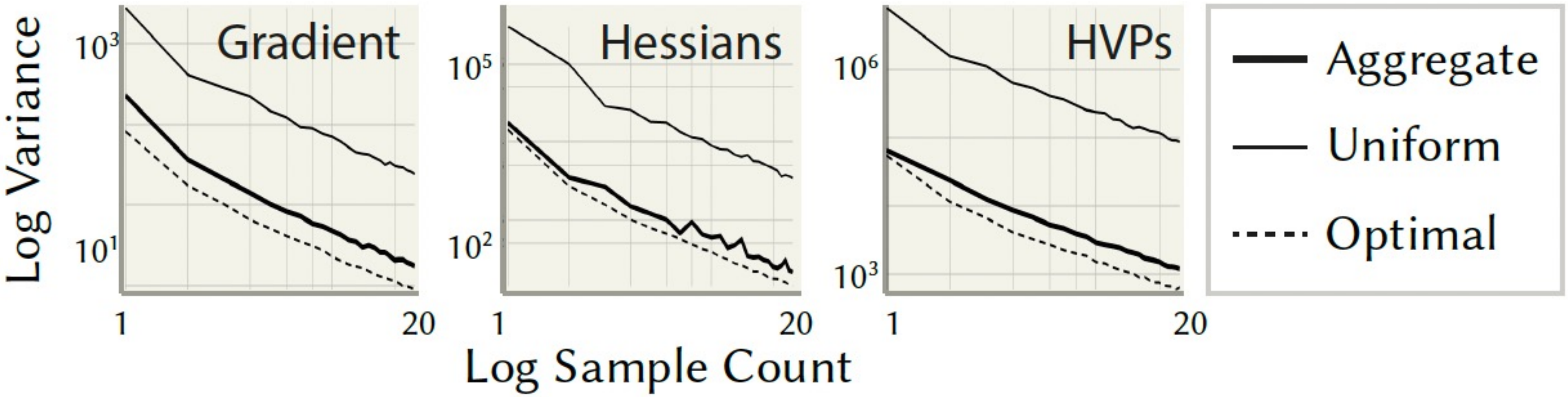
Ground truth

Conclusion & limitations

- Unbiased estimator for second order gradients
- With conjugate gradient optimizer
- Smooth out plateaus
- Works with black box functions

Conclusion & limitations

- Variance–cost trade-off
- Full Hessians scale poorly in very high-D
- Needs PSD safeguards (Hessian modification/trust region)



Stochastic gradient estimation for higher-order differentiable rendering

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- Project page: https://wangzican.github.io/publication/hodr

Project page