

A Linear N-Point Solver for Structure and Motion from Asynchronous Tracks

ICCV 2025 Highlight

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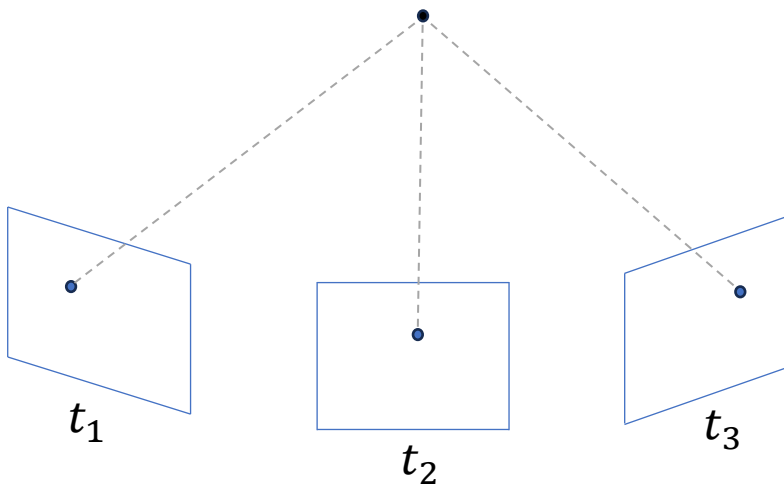


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Challenge

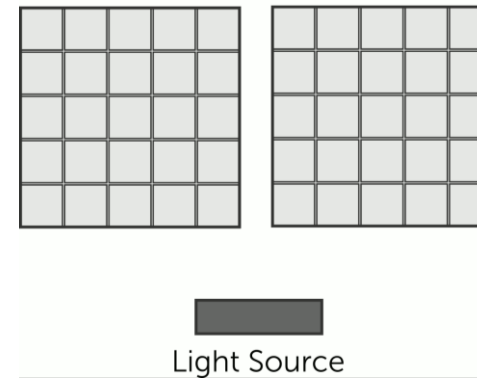
Traditional Methods



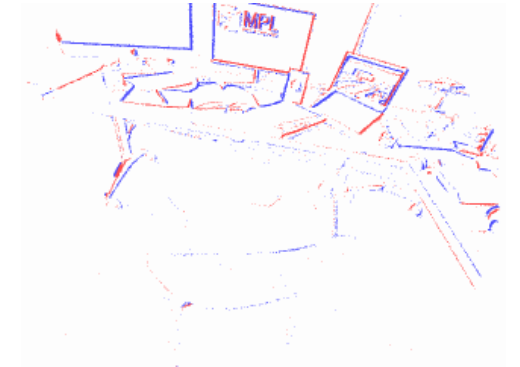
Classic solvers like the 5-point or 8-point algorithm require point track from **synchronized** views

Asynchronous Data

Rolling Shutter



Event Cameras



This assumption breaks down for rolling shutter and event cameras, which produce **asynchronous** data

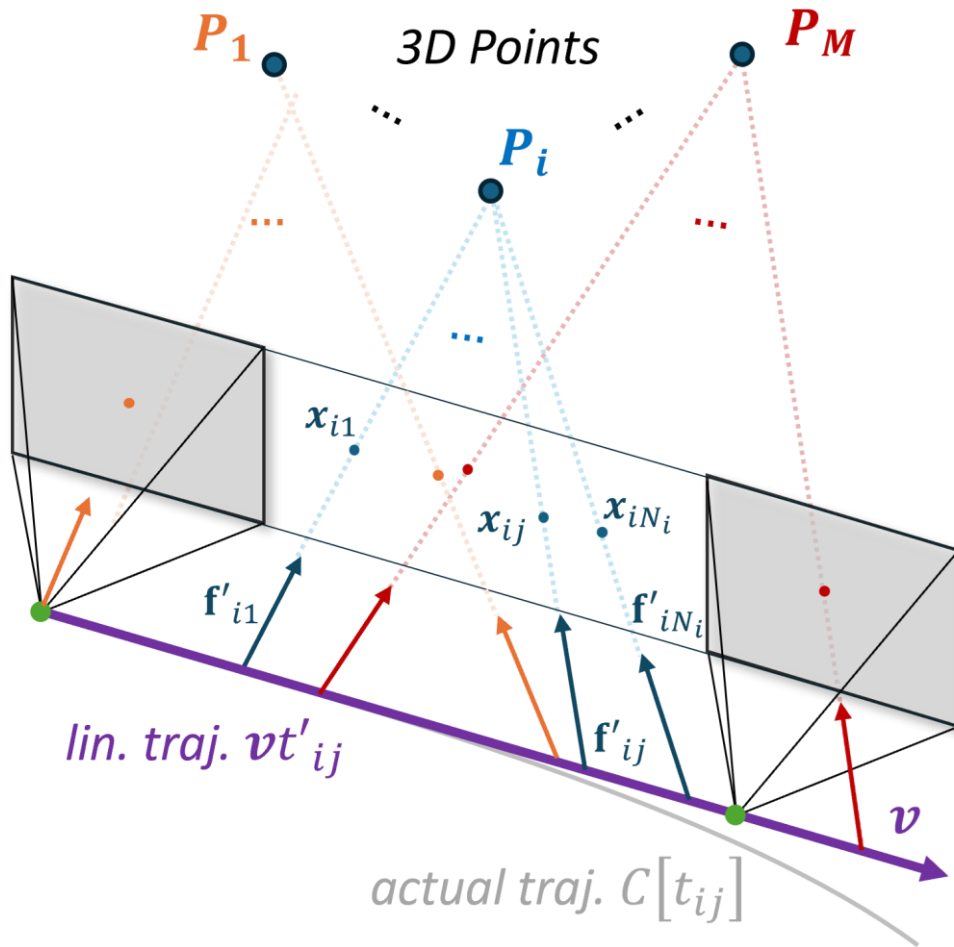
Our Goal

Develop a unified solver to estimate 3D structure and velocity directly from these asynchronous point tracks

Contributions

- A novel linear N-Point solver that uses an arbitrary number of **asynchronous** point feature tracks
- A general formulation that handles **various** sensors like global shutter, rolling shutter, and event cameras – and can even combine them
- A highly **efficient** method that employs Schur complement to reduce time complexity.

From Geometry to Linear System



1. Single 2D Observation

$$\mathbf{f}_{ij} = \mathbf{K}^{-1} \tilde{\mathbf{x}}_{ij}$$

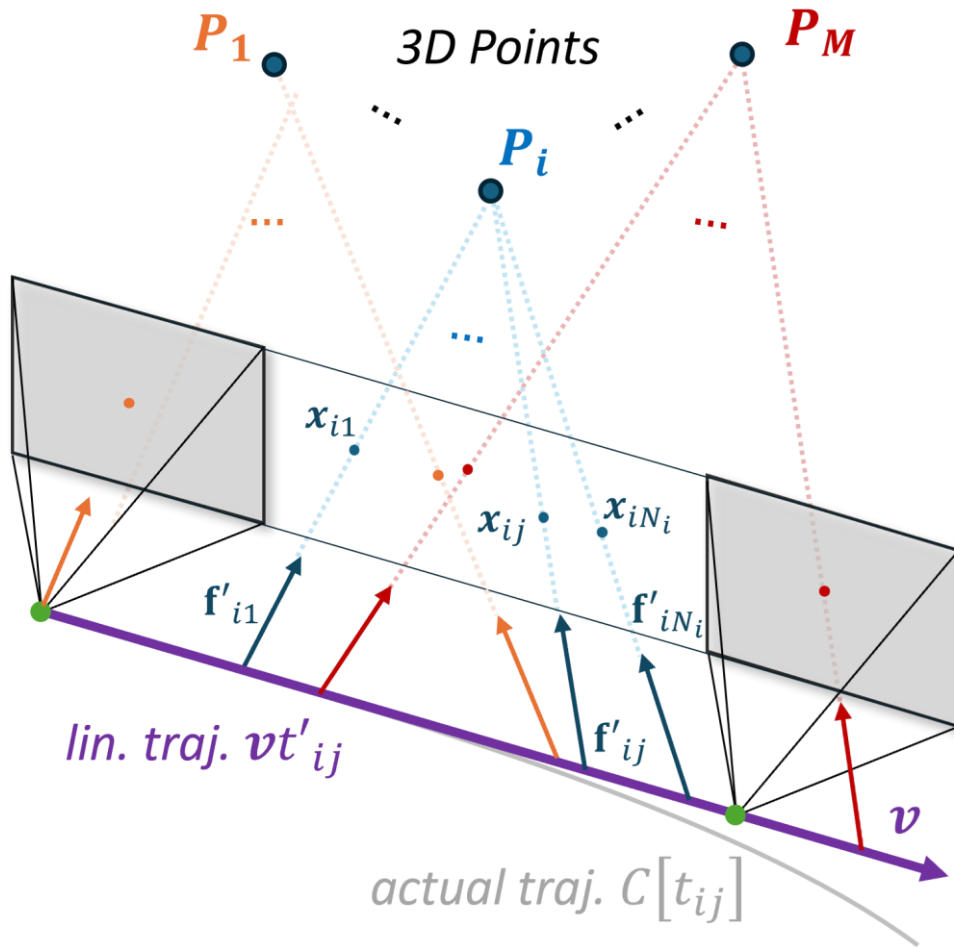
$$\mathbf{f}_{ij} \times (\mathbf{R}^\top(t_{ij}) (\mathbf{P}_i - \mathbf{p}(t_{ij}))) = \mathbf{0}_{3 \times 1}$$

Bearing
vector
Rotation
3D
Point
Camera
Position

$$[\mathbf{f}'_{ij}]_{\times} \mathbf{P}_i - t'_{ij} [\mathbf{f}'_{ij}]_{\times} \mathbf{v} = \mathbf{0}_{3 \times 1}$$

Applying constant velocity motion model

From Geometry to Linear System

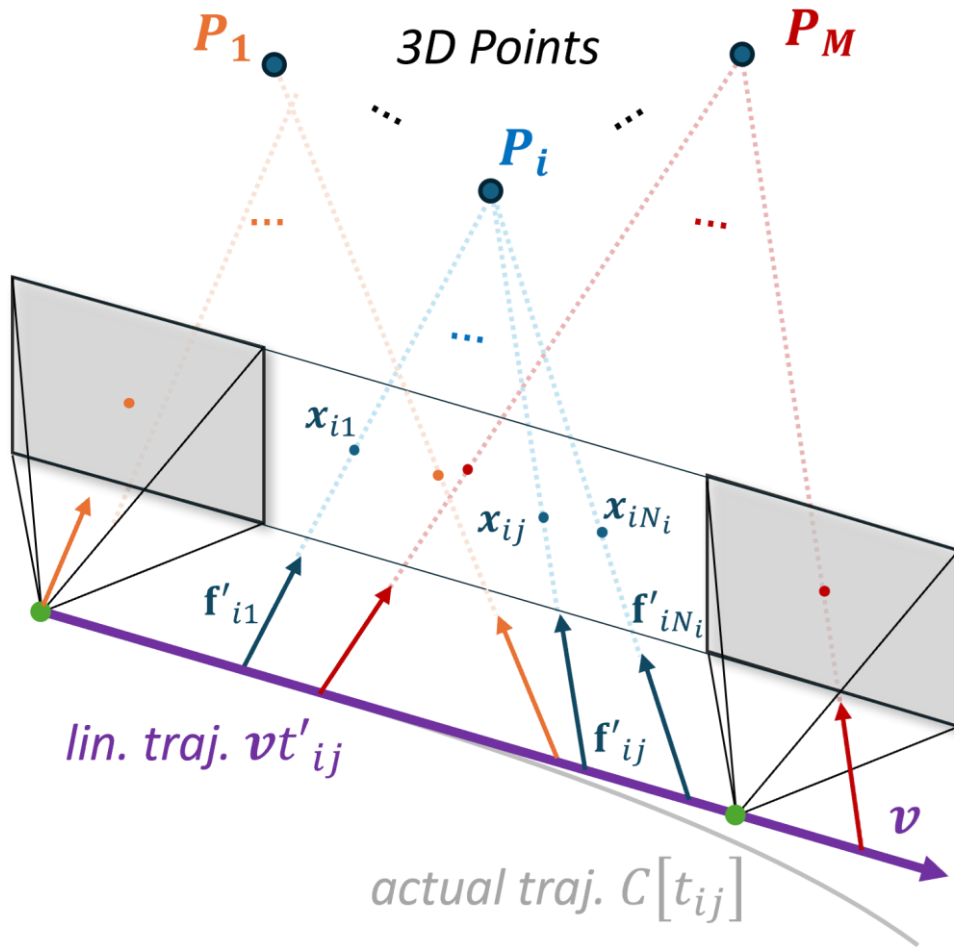


2. Multiple observations from a single 3D point

$$\underbrace{\begin{bmatrix} [\mathbf{f}'_{i1}]_{\times} & -t'_{i1} [\mathbf{f}'_{i1}]_{\times} \\ \vdots & \vdots \\ [\mathbf{f}'_{iN_i}]_{\times} & -t'_{iN_i} [\mathbf{f}'_{iN_i}]_{\times} \end{bmatrix}}_{\doteq [\mathbf{F}_i \quad \mathbf{G}_i] \in \mathbb{R}^{3N_i \times 6}} \begin{bmatrix} \mathbf{P}_i \\ \mathbf{v} \end{bmatrix} = \mathbf{0}_{3N_i \times 1}$$

Different timestamps to avoid degeneracy

From Geometry to Linear System

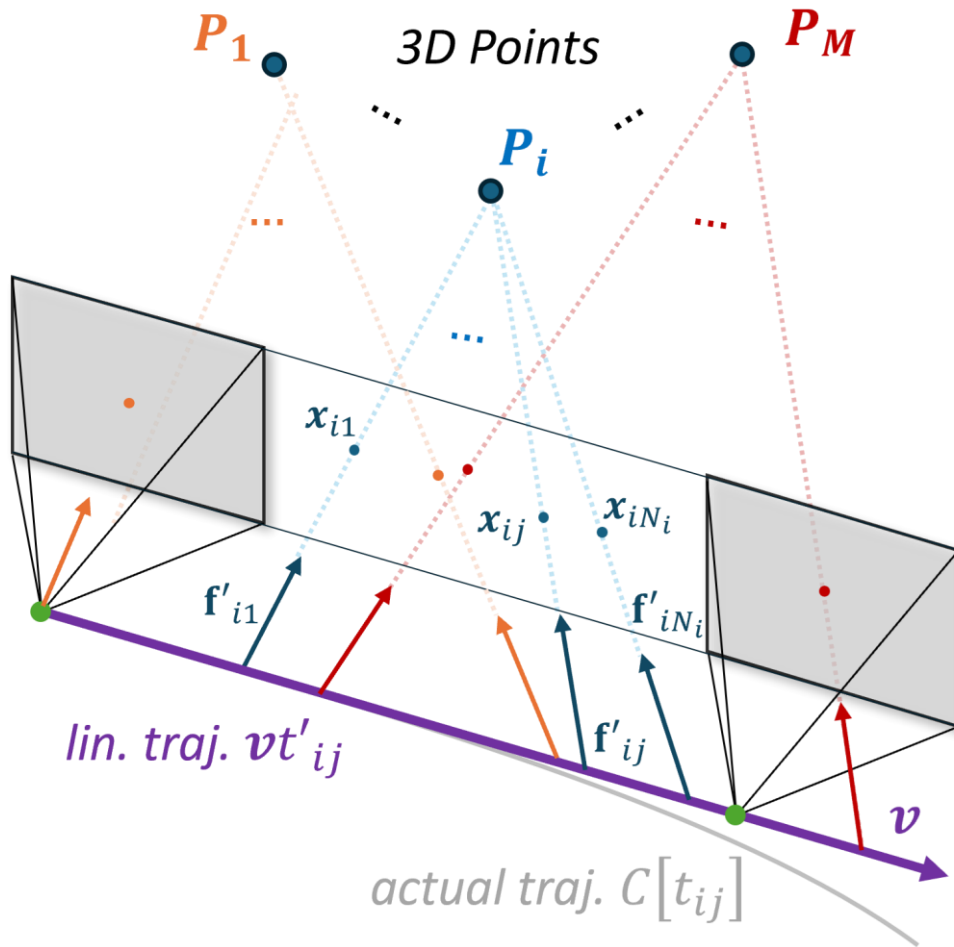


3. Multiple 3D points

$$\underbrace{\begin{bmatrix} \mathbf{F}_1 & & & \mathbf{G}_1 \\ & \mathbf{F}_2 & & \mathbf{G}_2 \\ & & \ddots & \vdots \\ & & & \mathbf{F}_M & \mathbf{G}_M \end{bmatrix}}_{\doteq \mathbf{A} \in \mathbb{R}^{3N \times (3M+3)}} \underbrace{\begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \vdots \\ \mathbf{P}_M \\ \mathbf{v} \end{bmatrix}}_{\doteq \mathbf{x} \in \mathbb{R}^{3M+3}} = \mathbf{0}_{3N \times 1}$$

Stacking constraints from multiple tracks

From Geometry to Linear System



4. Schur Complement

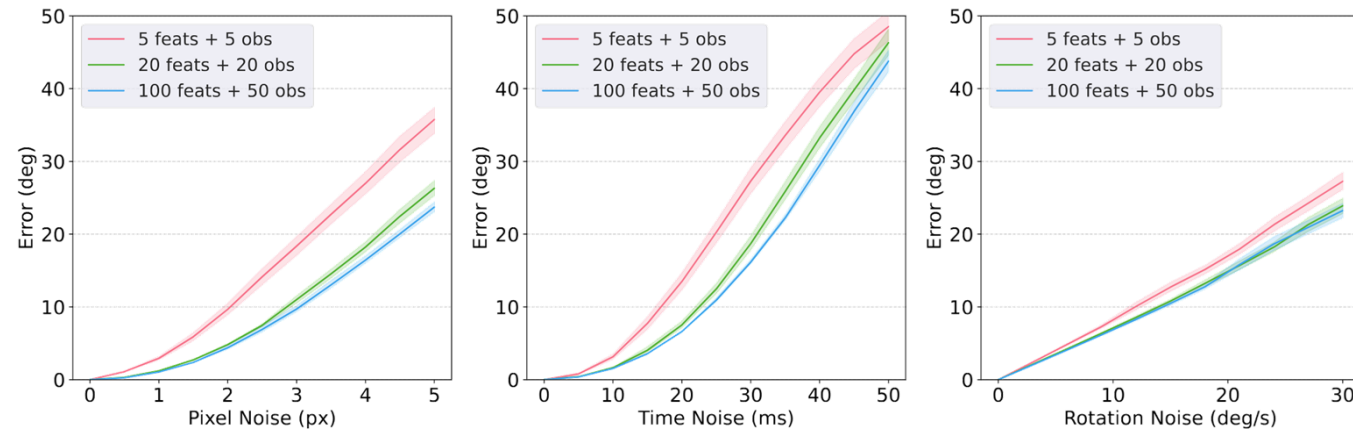
$$\underbrace{\mathbf{A}^\top \mathbf{A}}_{\doteq \mathbf{M}} \mathbf{x} = \begin{bmatrix} \mathbf{M}_A & \mathbf{M}_B \\ \mathbf{M}_B^\top & \mathbf{M}_D \end{bmatrix} \begin{bmatrix} \mathbf{P}_{1:M} \\ \mathbf{v} \end{bmatrix} = \mathbf{0}_{(3M+3) \times 1}$$

$$\underbrace{(\mathbf{M}_D - \mathbf{M}_B^\top \mathbf{M}_A^{-1} \mathbf{M}_B)}_{\doteq \mathbf{B} \in \mathbb{R}^{3 \times 3}} \mathbf{v} = \mathbf{0}$$

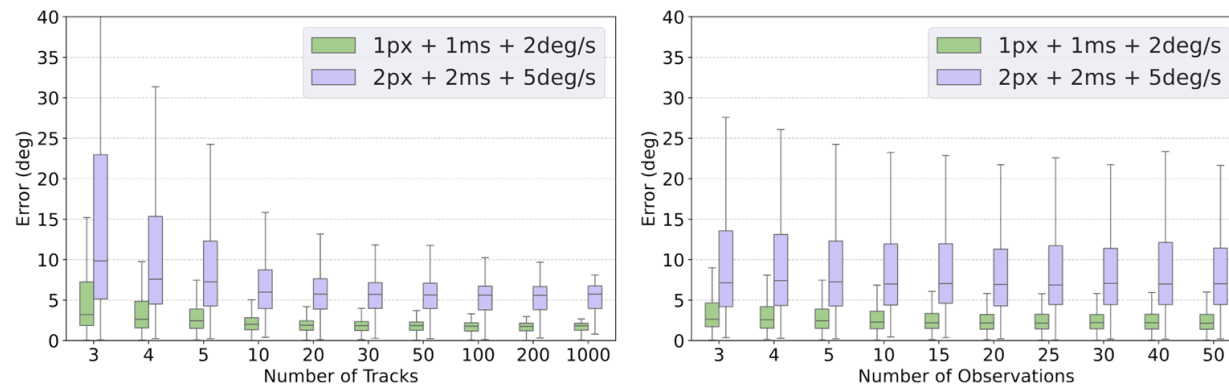
Only solve for velocity via SVD on B

Key Results: Robustness

Noise Resilience



Impacts of Spatial-temporal observations



Key Results: Real Performance

Global Shutter

VECtor dataset

Seq.	eventail [11] + E	Ours	Ours*
<i>desk-normal</i>	22.7 / 23.4	15.1 / 8.5	10.2 / 7.3
<i>sofa-normal</i>	21.9 / 17.6	15.9 / 7.8	9.8 / 6.3
<i>mountain-normal</i>	25.2 / 21.4	17.1 / 7.5	10.9 / 6.1
<i>shapes_translation</i>	31.8 / 32.7	17.1 / 7.2	9.9 / 6.2
<i>boxes_translation</i>	34.8 / 34.1	16.5 / 11.6	13.3 / 10.7

Error metric



$$\theta_{\text{err}} = \arccos \left(\frac{\mathbf{v}_{\text{gt}}^T \hat{\mathbf{v}}}{\|\mathbf{v}_{\text{true}}\| \|\hat{\mathbf{v}}\|} \right)$$

Rolling Shutter

TUM dataset

Seq.	with correction			no correction	
	eventail [11] + RS	Ours	Ours*	Ours	Ours*
<i>Seq 4</i>	43.8 / 40.8	27.5 / 20.1	22.6 / 17.4	28.1 / 22.9	22.8 / 15.7
<i>Seq 5</i>	45.5 / 44.8	24.7 / 17.0	19.3 / 13.8	27.0 / 18.4	19.2 / 14.6

- Note: results are mean/median velocity error in degrees
- Ours* indicates results on subset with track inlier ratio > 0.9

Events

VECtor dataset

Seq.	eventail [11] + E	Ours + E	Ours* + E	Ours + E + GS	Ours* + E + GS
<i>desk-normal</i>	22.7 / 23.4	19.3 / 17.8	14.2 / 14.2	–	–
<i>sofa-normal</i>	21.9 / 17.6	19.0 / 18.5	16.3 / 14.9	–	–
<i>mountain-normal</i>	25.2 / 21.4	17.1 / 16.1	16.9 / 15.8	–	–
<i>shapes_translation</i>	31.8 / 32.7	16.8 / 10.1	13.0 / 9.1	14.4 / 7.5	7.0 / 6.7
<i>boxes_translation</i>	34.3 / 34.1	12.6 / 10.0	12.1 / 7.7	10.3 / 8.1	9.3 / 5.9

Thanks for watching

Join me at my ICCV Poster Session!



Date: Tuesday, October 21st



Time: 11:45 a.m. — 1:45 p.m. HST



Location: Poster #407

