

EQUICAPS: PREDICTOR-FREE POSE-AWARE PRE-TRAINED CAPSULE NETWORKS



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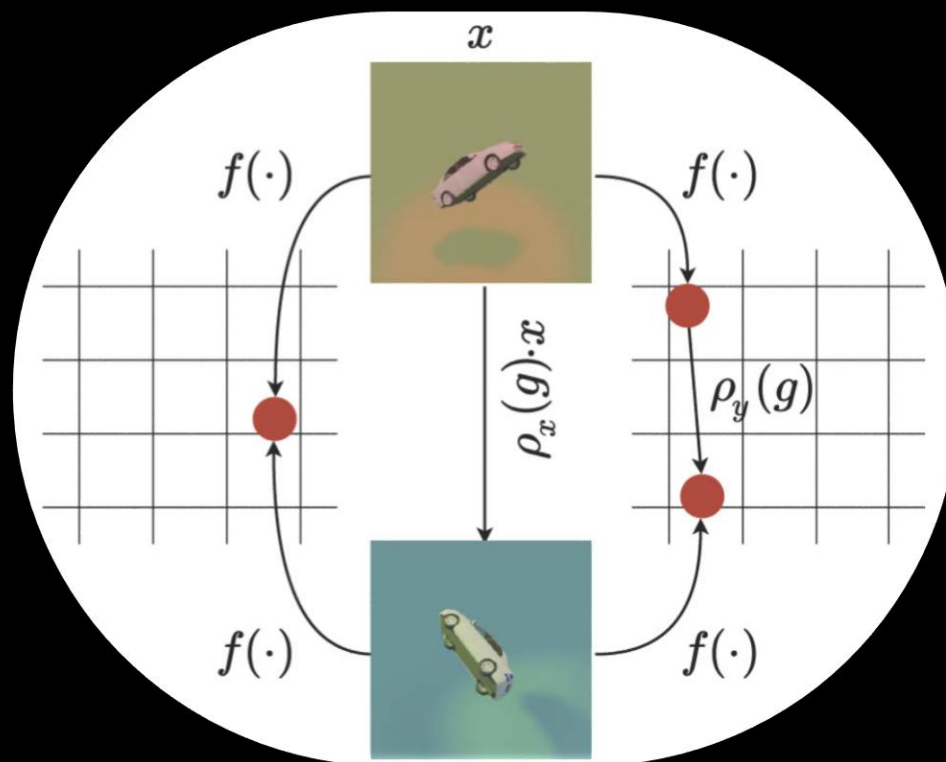


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INVARIANT VS. EQUIVARIANT SELF-SUPERVISION



Invariance

$$\mathcal{L}_{inv} = \mathcal{L}(f(p_x(g) \cdot x), f(x))$$

Equivariance

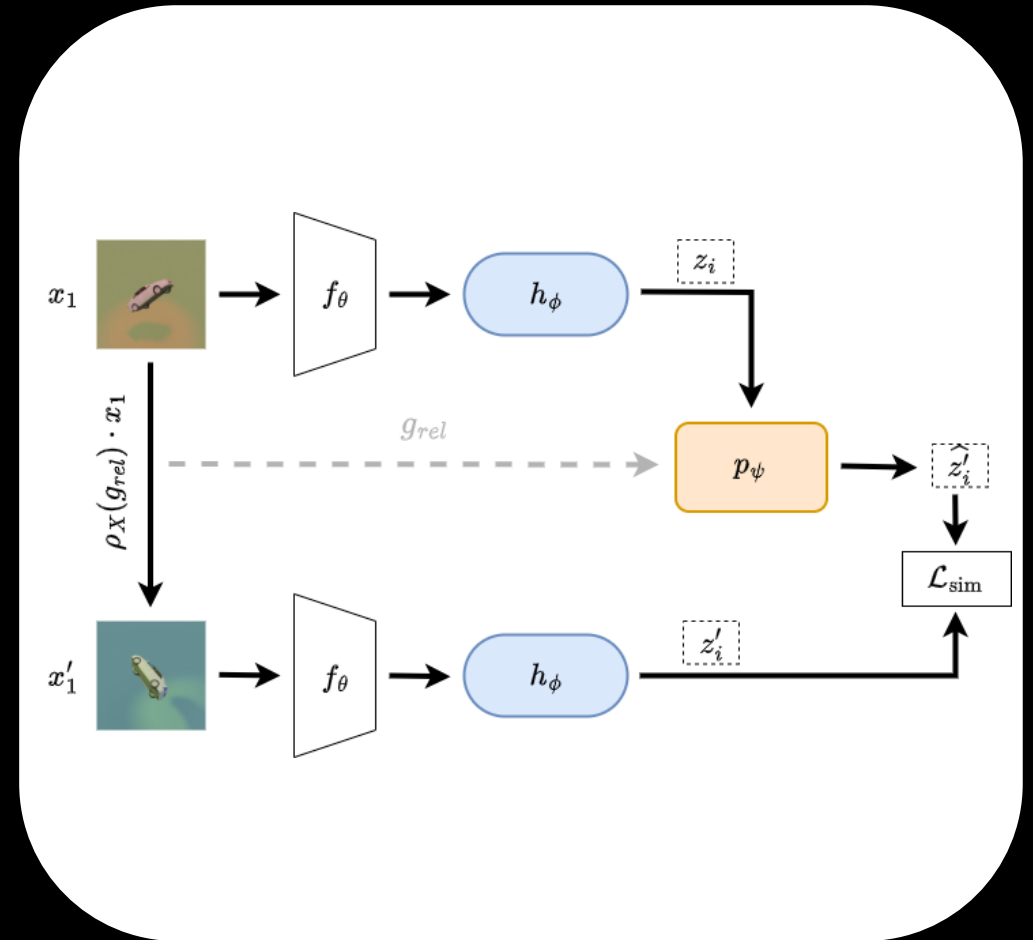
$$\mathcal{L}_{equi} = \mathcal{L}(f(p_x(g) \cdot x), p_y(g) \cdot f(x))$$

Visual recognition involves not only identifying **what** an object is but also understanding **how** it is presented [1].

MOTIVATION

Most equivariant SSL (e.g., SIE, EquiMod) enforce equivariance via objective functions/predictor.

- **Few exploit equivariant architectures in SSL.**
- They use a **predictor** p_ψ s.t. $\hat{z}_i' = p_\psi(z_i, g_{rel})$,
- produce ad hoc representations that are **hard to interpret and manipulate**,
- rely on architectures (e.g., CNNs) that are **not naturally equivariant**, and
- add **extra complexity** via extra modules.



LEVERAGE CAPSULE NETWORKS' INDUCTIVE BIASES

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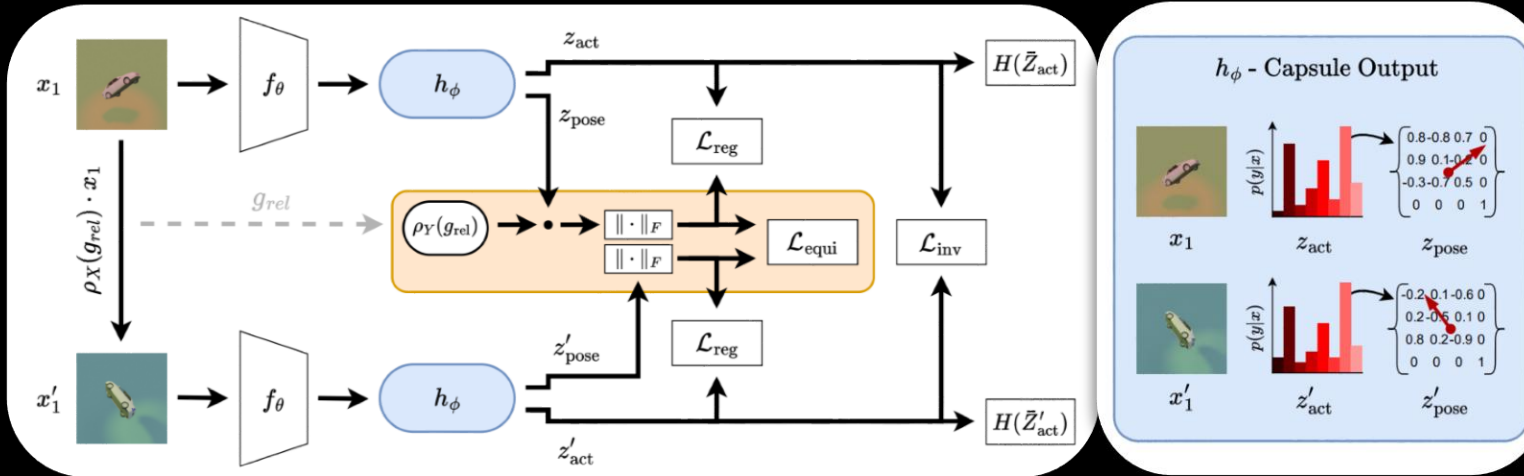
- Few exploit equivariant architectures in SSL.
- They use a predictor p_ψ s.t. $\hat{z} = p_\psi(z', g_{rel})$,
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Inductive
biases

Using routing based on agreement of **part-whole relationships**, naturally encode both:

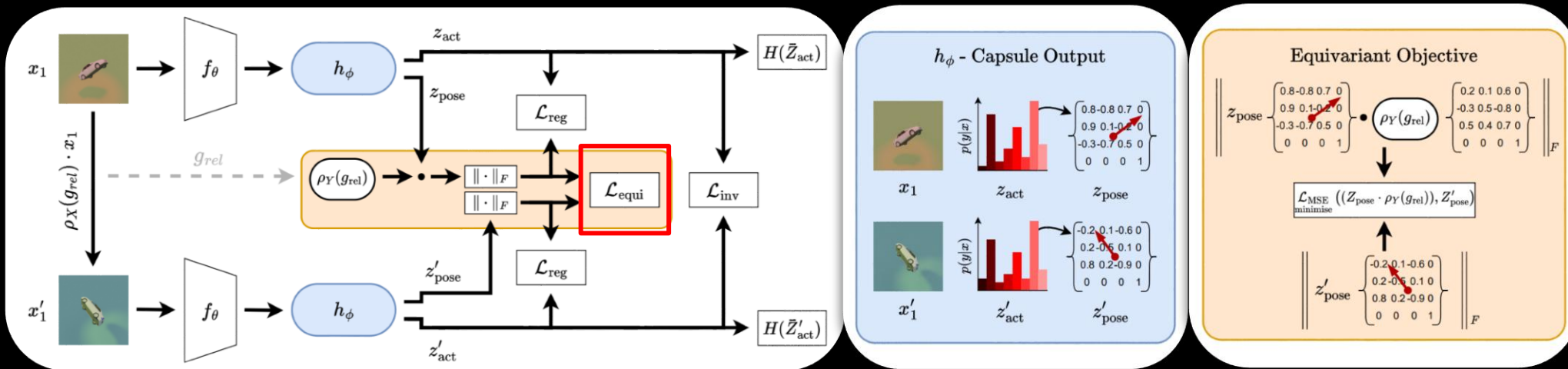
- the existence of an entity (**invariance**), and
 - its instantiation parameters (**equivariance**).
-
- **We directly** leverage capsules' equivariant properties,
 - gain intuitive **control and interpretability** of the representations (4x4 pose matrices),
 - **and keep a streamlined framework.**

EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



- To reduce computation, rely on the **non-iterative** self-routing [2] algorithm.
- The activation vectors encode **transformation-invariant** properties
- The pose matrices capture **transformation-equivariant** properties.

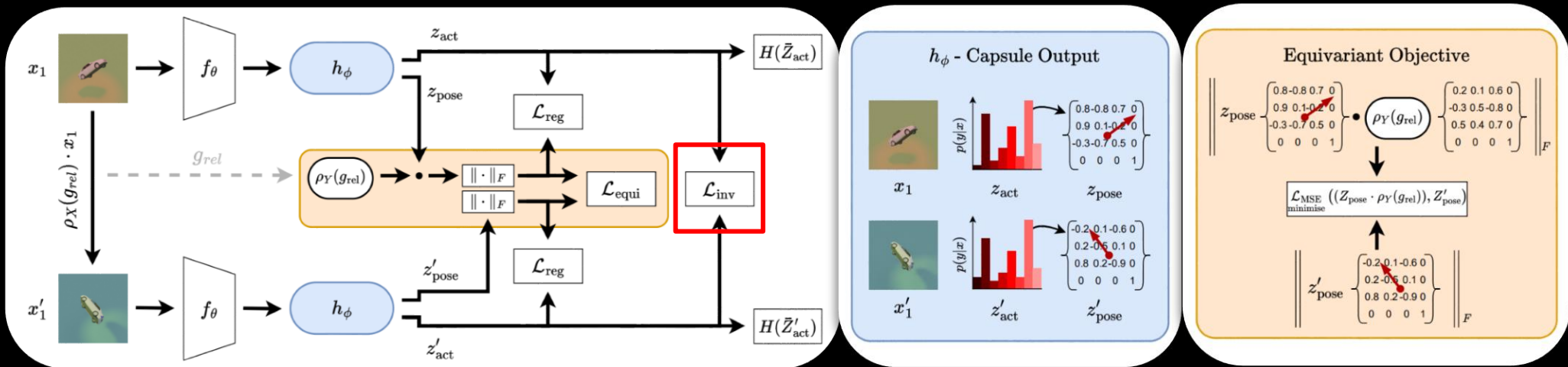
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$$\mathcal{L}_{equi} = \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_Y(g_{rel,i})}{\|Z_{i,pose} \cdot p_Y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2.$$

This direct manipulation in the latent space removes the need for a predictor.

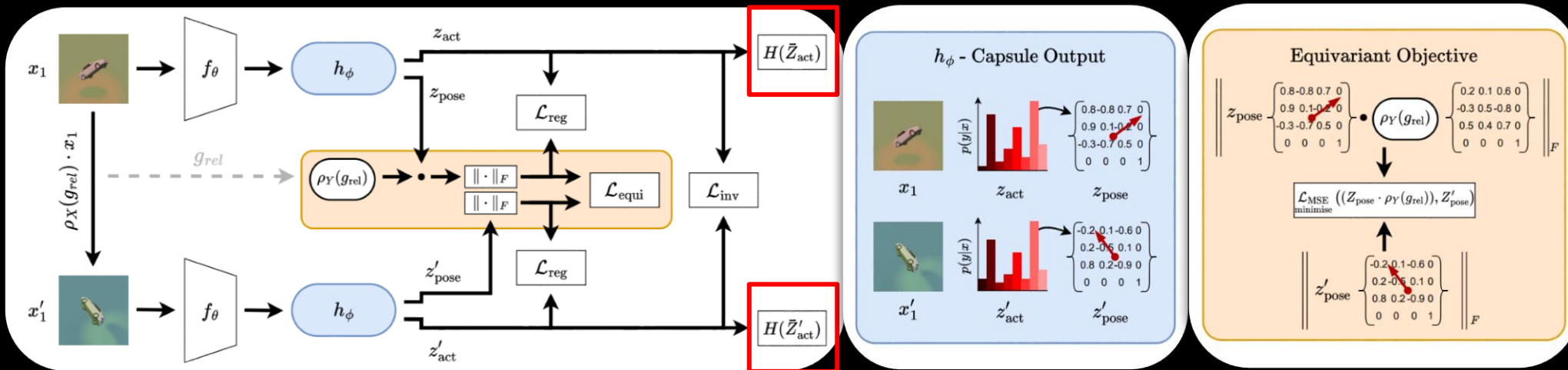
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$$\mathcal{L}_{inv} = H(Z_{act}, Z'_{act}).$$

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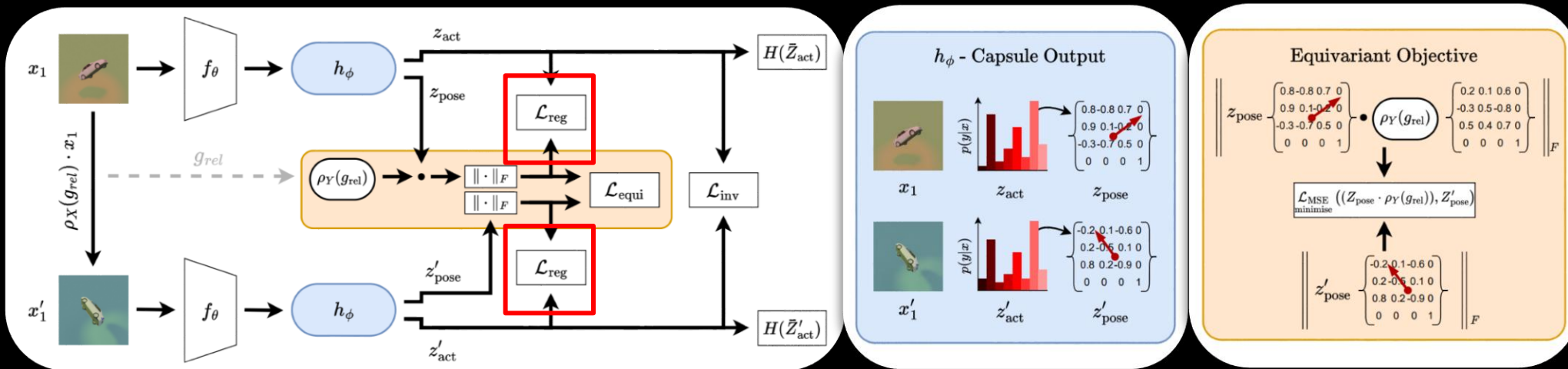


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$$\mathcal{L}_{ME-MAX} = H(\bar{Z}_{act}) + H(\bar{Z}'_{act}).$$

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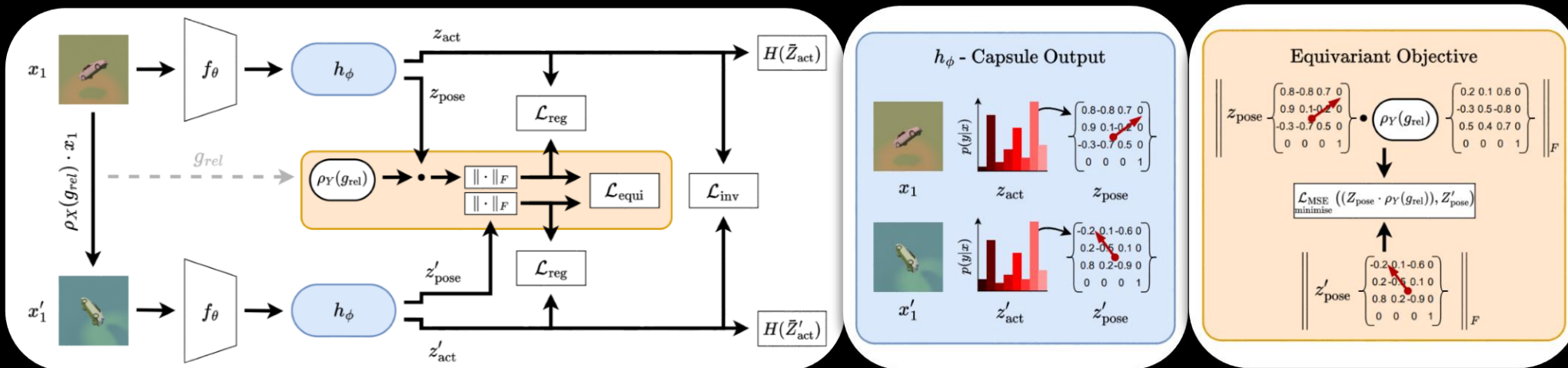
$$\mathcal{L}_{ME-MAX} = H(\bar{Z}_{act}) + H(\bar{Z}'_{act}).$$

$$\mathcal{L}_{reg}(Z_{cat}) = \lambda_V V(Z_{cat}) + \lambda_C C(Z_{cat}) \text{ where}$$

$$V(Z_{cat}) = \frac{1}{d} \sum_{j=1}^d \max(0, 1 - \sqrt{\text{Var}(Z_{cat \cdot j})}),$$

$$C(Z_{cat}) = \frac{1}{d} \sum_{i \neq j} \text{Cov}(Z_{cat})_{i,j}^2.$$

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The overall loss is a **combination**:

$$\begin{aligned} \mathcal{L}_{EquiCaps} = & \lambda_{inv} H(Z_{act}, Z'_{act}) + H(\bar{Z}_{act}) + H(\bar{Z}'_{act}) \\ & + \lambda_{equi} \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot \rho_Y(g_{rel,i})}{\|Z_{i,pose} \cdot \rho_Y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2 \\ & + \mathcal{L}_{reg}(Z_{cat}) + \mathcal{L}_{reg}(Z'_{cat}). \end{aligned}$$

EquiCaps can theoretically handle **any transformation** which can be expressed as **a matrix** without architectural changes.

3DIEBENCH-T: INVARIANT-EQUIVARIANT BENCHMARK

- Extends 3DIEBench from $SO(3)$ to **$SE(3)$** , increasing task complexity.
- Comprises:
 - **2,623,600** images
 - **55** classes
 - rendered from **52,472** ShapeNetCoreV2 3D models
 - under **50** (simultaneous $SE(3)$ + colour) transformations per model.



QUANTITATIVE RESULTS

Pre-train for rotation equivariance only

| Method | Classification (Top-1) | | Rotation (R^2) | | Translation (R^2) | Colour (R^2) | |
|---|------------------------|--------------|--------------------|-------------|-----------------------|------------------|-------------|
| | 3DIEBench | 3DIEBench-T | 3DIEBench | 3DIEBench-T | 3DIEBench-T | 3DIEBench | 3DIEBench-T |
| <i>Supervised Methods</i> | | | | | | | |
| ResNet-18 | 86.45 | 80.13 | 0.77 | 0.73 | 0.67 | 0.99 | 0.99 |
| <i>Invariant and Parameter Prediction Methods</i> | | | | | | | |
| VICReg | 84.28 | 74.71 | 0.45 | 0.39 | 0.22 | 0.10 | 0.50 |
| SimCLR | 86.73 | 80.08 | 0.52 | 0.44 | 0.25 | 0.29 | 0.50 |
| SimCLR + AugSelf | 87.44 | 80.86 | 0.75 | 0.69 | 0.50 | 0.28 | 0.51 |
| <i>Equivariant Methods</i> | | | | | | | |
| SEN | 86.99 | 80.20 | 0.51 | 0.45 | 0.26 | 0.29 | 0.47 |
| EquiMod | 87.39 | 80.76 | 0.50 | 0.43 | 0.24 | 0.29 | 0.38 |
| SIE | 82.94 | 75.56 | 0.73 | 0.45 | 0.20 | 0.07 | 0.46 |
| CapsIE | 79.14 | 75.20 | 0.74 | 0.60 | 0.46 | 0.01 | 0.03 |
| EquiCaps | 83.24 | 76.91 | 0.78 | 0.73 | 0.60 | 0.09 | 0.05 |

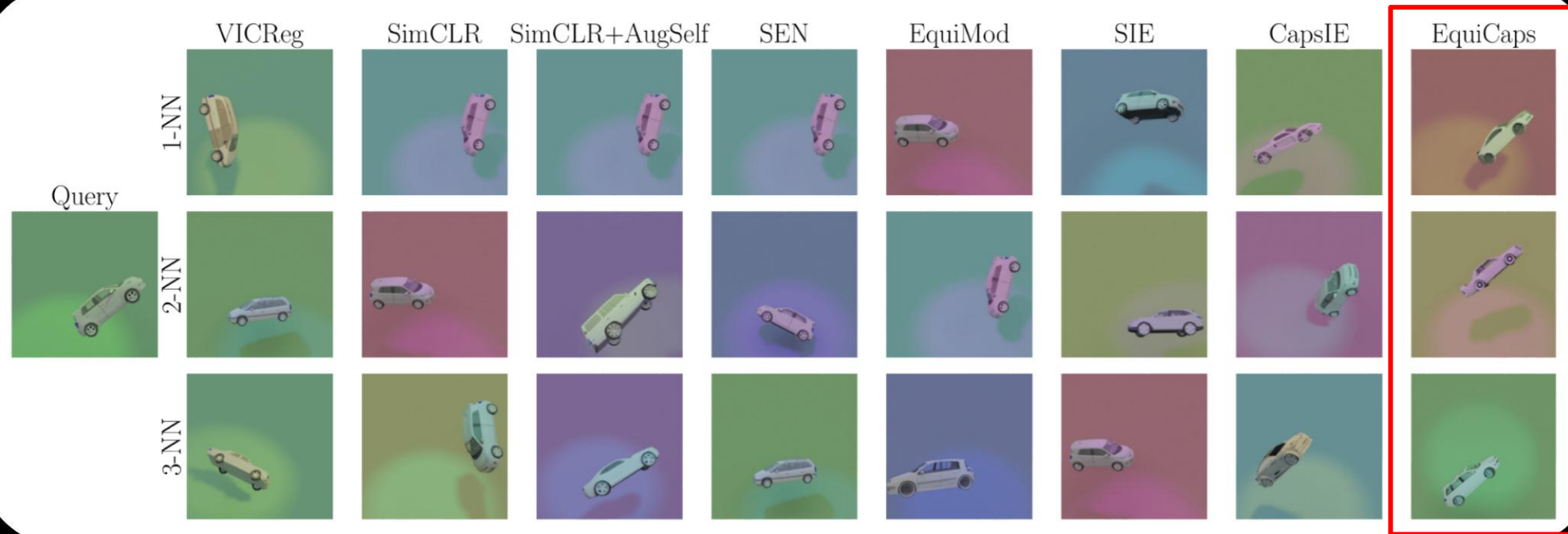
QUANTITATIVE RESULTS

Pre-train for rotation & translation equivariance

| Method | Classification (Top-1) | Rotation (R^2) | Translation (R^2) | Colour (R^2) |
|------------------|------------------------------|-------------------------------|-----------------------------|------------------------|
| SimCLR + AugSelf | 81.04 \uparrow 0.18 | 0.69 = 0.00 | 0.64 \uparrow 0.14 | 0.51 = 0.00 |
| SEN | 80.23 \uparrow 0.03 | 0.46 \uparrow 0.01 | 0.28 \uparrow 0.02 | 0.50 \uparrow 0.03 |
| EquiMod | 80.89 \uparrow 0.13 | 0.46 \uparrow 0.03 | 0.37 \uparrow 0.13 | 0.37 \downarrow 0.01 |
| SIE | 75.91 \uparrow 0.35 | 0.48 \uparrow 0.03 | 0.22 \uparrow 0.02 | 0.36 \downarrow 0.10 |
| CapsIE | 76.31 \uparrow 1.11 | 0.62 \uparrow 0.02 | 0.53 \uparrow 0.07 | 0.03 = 0.00 |
| EquiCaps | 77.88 \uparrow 0.97 | 0.71 \downarrow 0.02 | 0.61 \uparrow 0.01 | 0.02 \downarrow 0.03 |

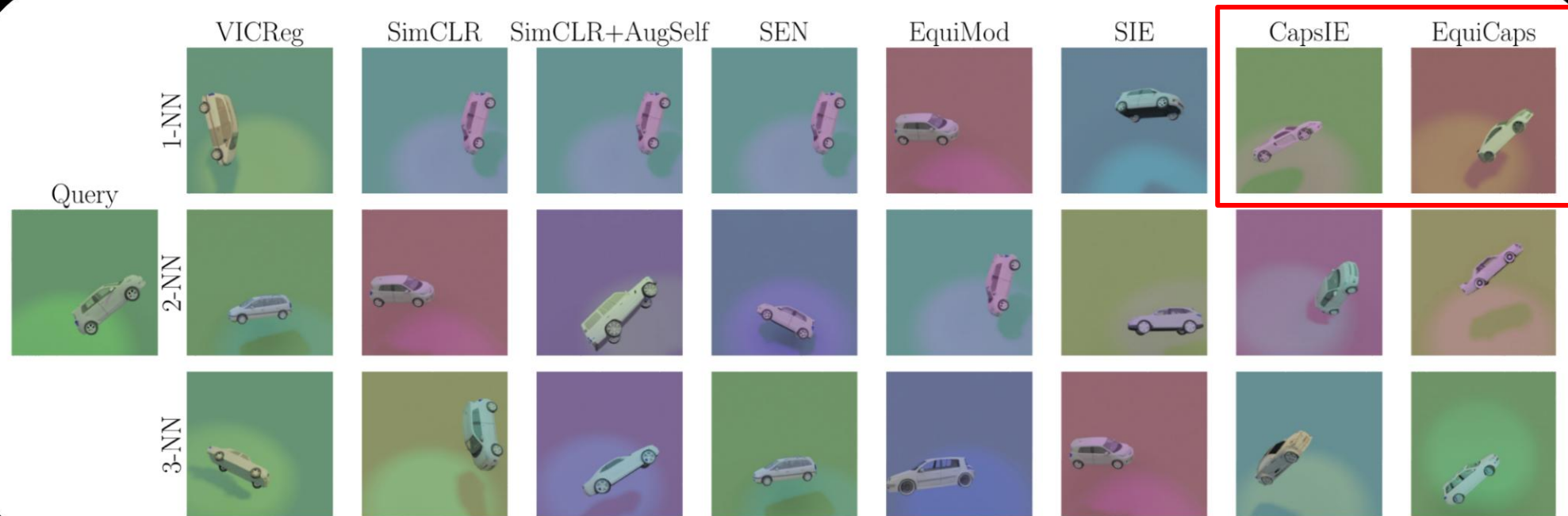
QUALITATIVE RESULTS

k -NN representation retrieval



QUALITATIVE RESULTS

k -NN representation retrieval



MAIN TAKEAWAYS

- **EquiCaps** (predictor-free equivariance)
 - Capsule-based projector
 - Controllable and interpretable latent space
- 3DIEBench-T (**SE(3)** benchmark)
- Extensive experiments
 - **SOTA** on rotation and translation prediction among the equivariant baselines
 - Capsule architectures show **improved generalisation** under combined SE(3) transformations and in transfer learning (including object detection)

THANK YOU



ArXiv



Code



Dataset