

# EQUICAPS: PREDICTOR-FREE POSE-AWARE PRE-TRAINED CAPSULE NETWORKS



Athinoulla Konstantinou  
University of Aberdeen



Georgios Leontidis  
University of Aberdeen

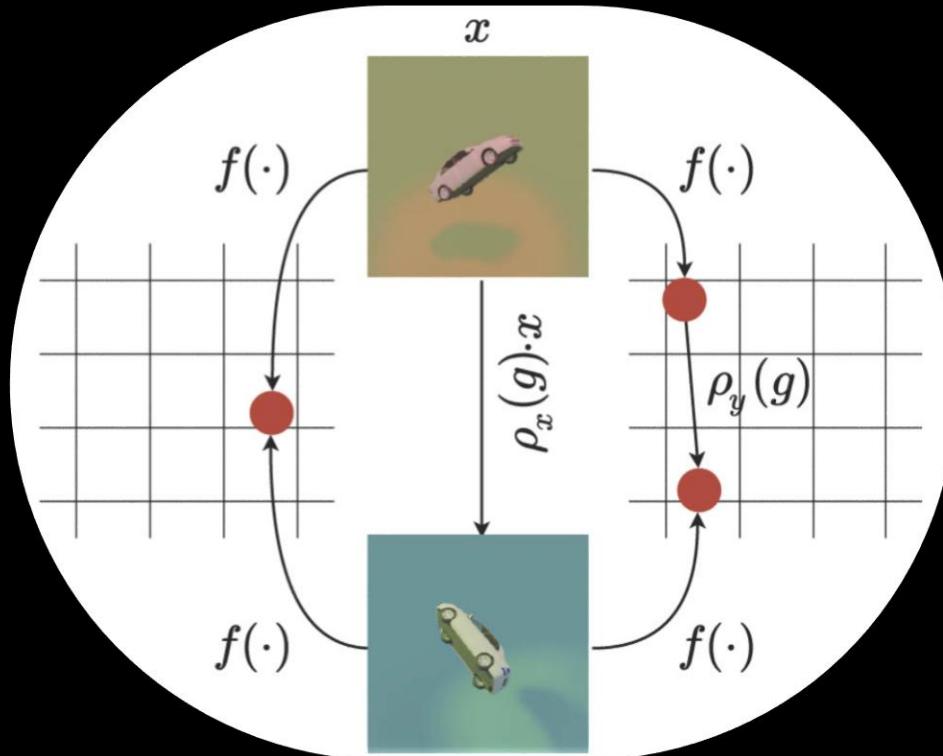


Mamatha Thota  
University of Lincoln



Aiden Durrant  
University of Aberdeen

# INVARIANT VS. EQUIVARIANT SELF-SUPERVISION



Invariance

$$\mathcal{L}_{inv} = \mathcal{L}\left(f(p_x(g) \cdot x), f(x)\right)$$

Equivariance

$$\mathcal{L}_{equi} = \mathcal{L}\left(f(p_x(g) \cdot x), p_y(g) \cdot f(x)\right)$$

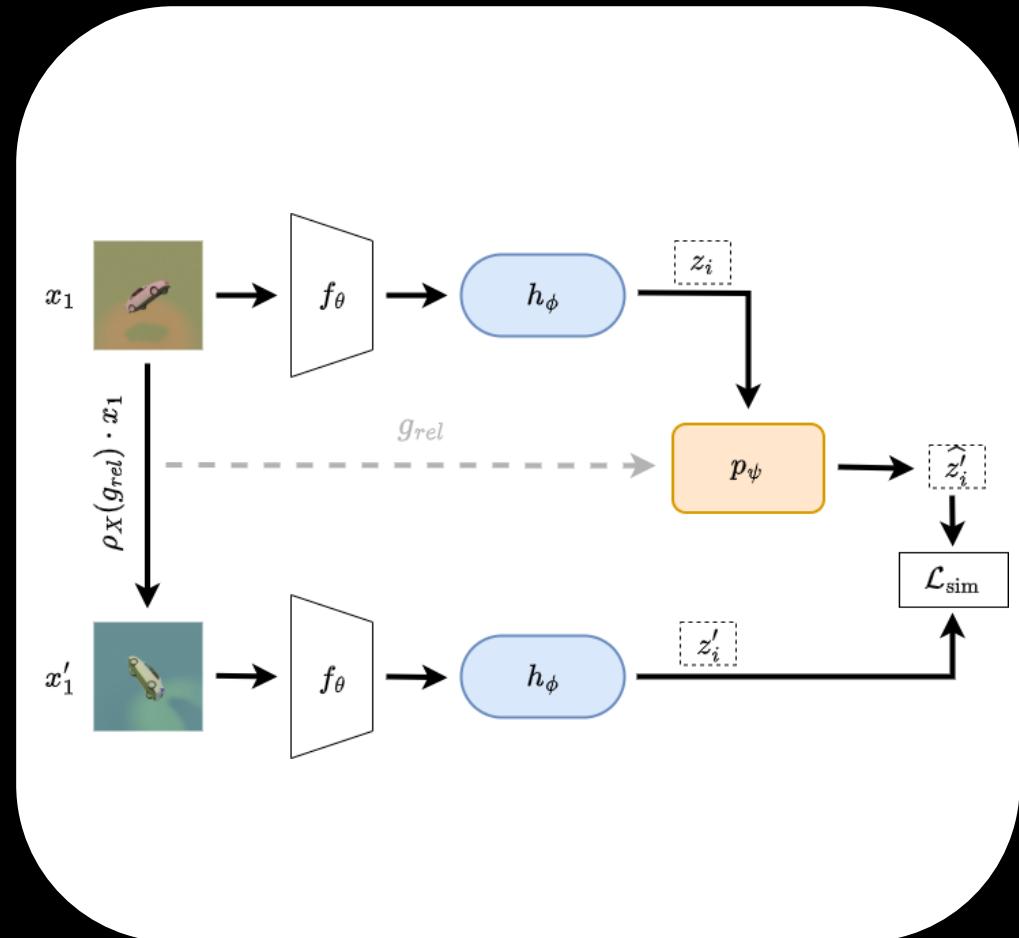
Visual recognition involves not only identifying **what** an object is but also understanding **how** it is presented [1].

2

# MOTIVATION

Most equivariant SSL (e.g., SIE, EquiMod) enforce equivariance via objective functions/predictor.

- **Few exploit equivariant architectures in SSL.**
- They use a **predictor**  $p_\psi$  s.t.  $\hat{z'_i} = p_\psi(z_i, g_{rel})$ ,
- produce ad hoc representations that are **hard to interpret and manipulate**,
- rely on architectures (e.g., CNNs) that are **not naturally equivariant**, and
- add **extra complexity** via extra modules.



# LEVERAGE CAPSULE NETWORKS' INDUCTIVE BIASES

Most equivariant SSL (e.g., SIE, EquiMod) enforce equivariance via objective functions/predictor.

- Few exploit equivariant architectures in SSL.
- They use a predictor  $p_\psi$  s.t.  $\hat{z} = p_\psi(z', g_{rel})$ .
- rely on architectures (e.g., CNNs) that are not naturally equivariant, and
- produce ad hoc representations that are hard to interpret and manipulate,
- add extra complexity via extra modules.

Inductive  
biases

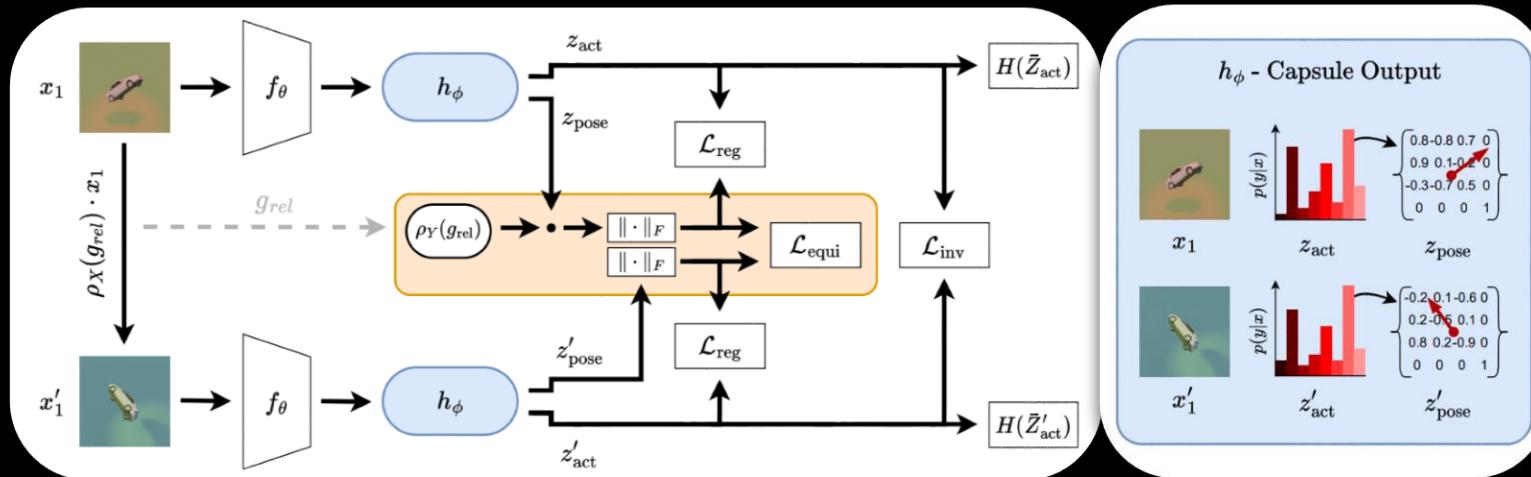
Using routing based on agreement of **part-whole relationships**, naturally encode both:

- the existence of an entity (**invariance**), and
- its instantiation parameters (**equivariance**).

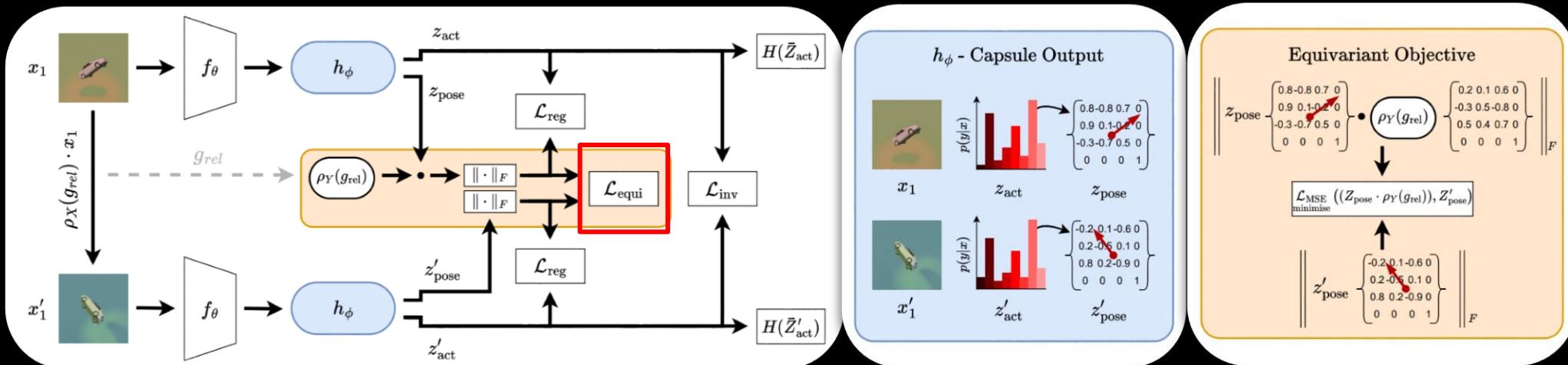
- **We directly** leverage capsules' equivariant properties,
- gain intuitive **control and interpretability** of the representations (4x4 pose matrices),
- **and keep a streamlined framework.**

# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



- To reduce computation, rely on the **non-iterative** self-routing [2] algorithm.
- The activation vectors encode **transformation-invariant** properties
- The pose matrices capture **transformation-equivariant** properties.

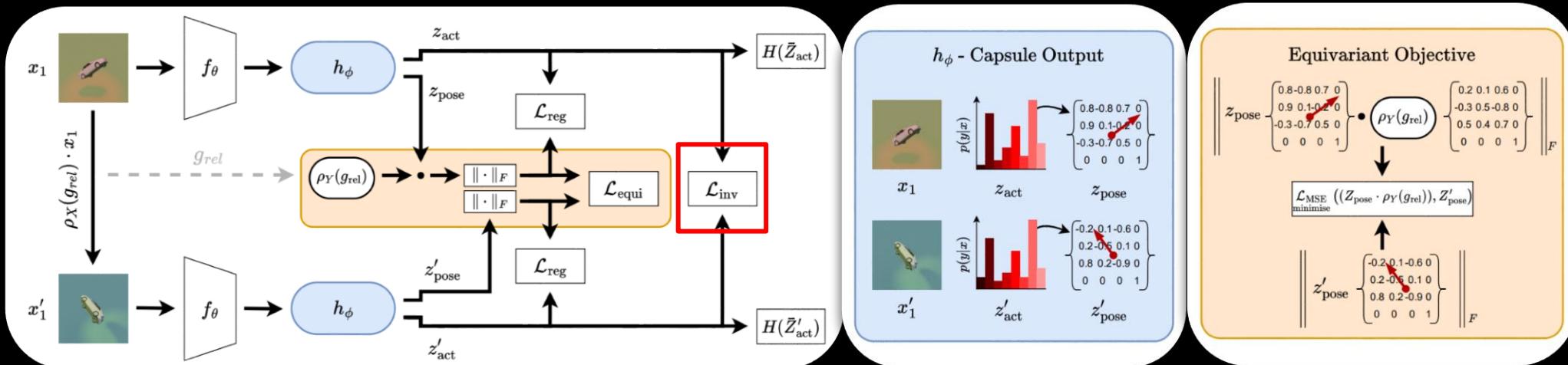
# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



$$\mathcal{L}_{equi} = \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_y(g_{rel,i})}{\|Z_{i,pose} \cdot p_y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2.$$

**This direct manipulation in the latent space removes the need for a predictor.**

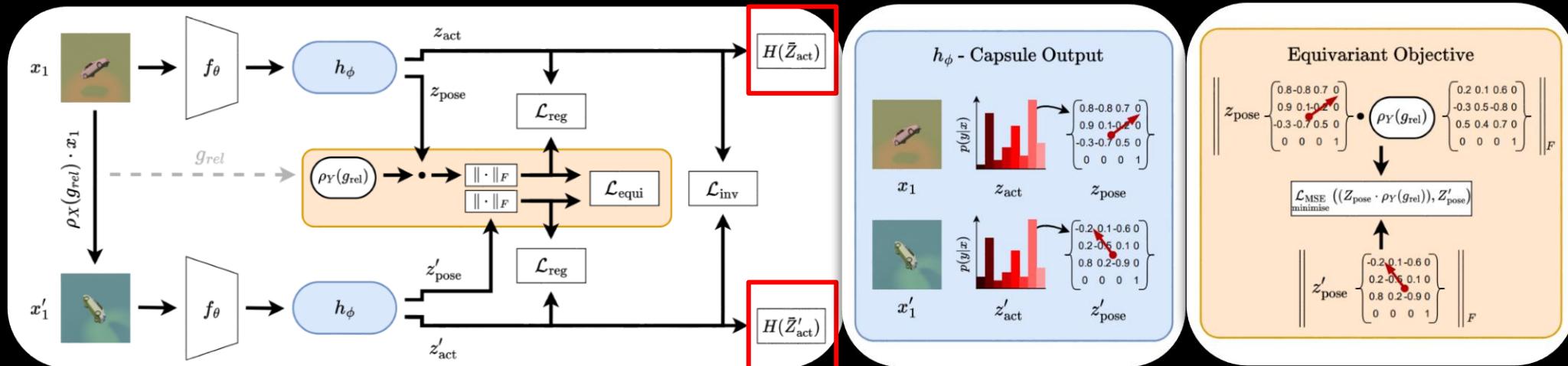
# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



$$\mathcal{L}_{equi} = \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_y(g_{rel,i})}{\|Z_{i,pose} \cdot p_y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2.$$

$$\mathcal{L}_{inv} = H(Z_{act}, Z'_{act}).$$

# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL

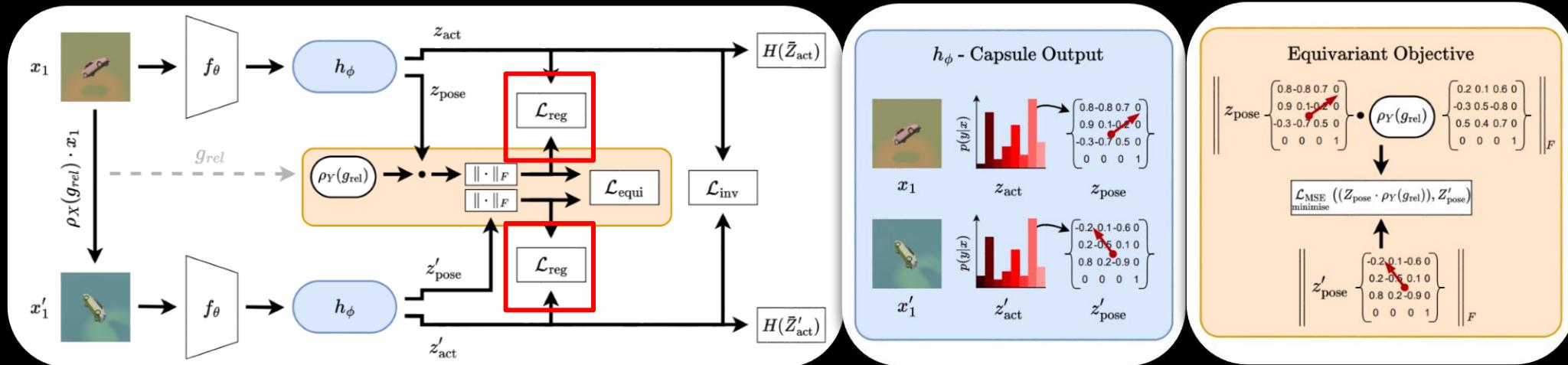


$$\mathcal{L}_{equi} = \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_y(g_{rel,i})}{\|Z_{i,pose} \cdot p_y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2.$$

$$\mathcal{L}_{inv} = H(Z_{act}, Z'_{act}).$$

$$\mathcal{L}_{ME-MAX} = H(\bar{Z}_{act}) + H(\bar{Z}'_{act}).$$

# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



$$\mathcal{L}_{equi} = \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_y(g_{rel,i})}{\|Z_{i,pose} \cdot p_y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2.$$

$$\mathcal{L}_{inv} = H(Z_{act}, Z'_{act}).$$

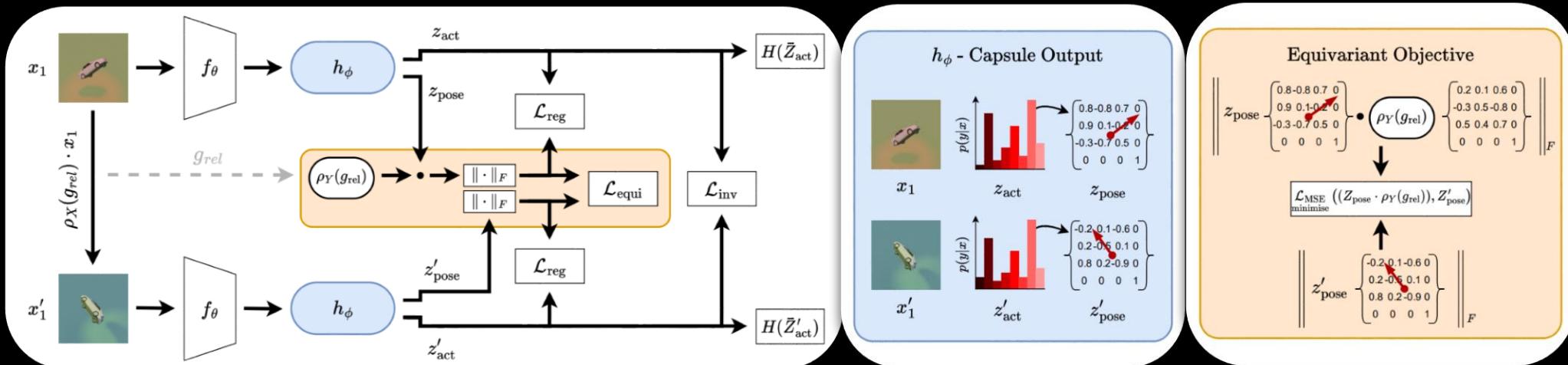
$$\mathcal{L}_{ME-MAX} = H(\bar{Z}_{act}) + H(\bar{Z}'_{act}).$$

$$\mathcal{L}_{reg}(Z_{cat}) = \lambda_V V(Z_{cat}) + \lambda_C C(Z_{cat}) \text{ where}$$

$$V(Z_{cat}) = \frac{1}{d} \sum_{j=1}^d \max(0, 1 - \sqrt{Var(Z_{cat \cdot j})}),$$

$$C(Z_{cat}) = \frac{1}{d} \sum_{i \neq j} Cov(Z_{cat})_{i,j}^2.$$

# EQUICAPS: PREDICTOR-FREE POSE-AWARE SSL



The overall loss is a **combination**:

$$\begin{aligned}
 \mathcal{L}_{EquiCaps} = & \lambda_{inv} H(Z_{act}, Z'_{act}) + H(\bar{Z}_{act}) + H(\bar{Z}'_{act}) \\
 & + \lambda_{equi} \frac{1}{B} \sum_{i=1}^B \left\| \frac{Z_{i,pose} \cdot p_y(g_{rel,i})}{\|Z_{i,pose} \cdot p_y(g_{rel,i})\|_F} - \frac{Z'_{i,pose}}{\|Z'_{i,pose}\|_F} \right\|_2^2 \\
 & + \mathcal{L}_{reg}(Z_{cat}) + \mathcal{L}_{reg}(Z'_{cat}).
 \end{aligned}$$

EquiCaps can theoretically handle **any transformation** which can be expressed as a **matrix** without architectural changes.

# 3DIEBENCH-T: INVARIANT-EQUIVARIANT BENCHMARK

- Extends 3DIEBench from  $SO(3)$  to **SE(3)**, increasing task complexity.
- Comprises:
  - **2,623,600** images
  - **55** classes
  - rendered from **52,472** ShapeNetCoreV2 3D models
  - under **50** (simultaneous  $SE(3)$  + colour) transformations per model.



# QUANTITATIVE RESULTS

Pre-train for rotation equivariance only

Method	Classification (Top-1)		Rotation ( $R^2$ )		Translation ( $R^2$ )		Colour ( $R^2$ )	
	3DIEBench	3DIEBench-T	3DIEBench	3DIEBench-T	3DIEBench-T	3DIEBench	3DIEBench-T	
<i>Supervised Methods</i>								
ResNet-18	86.45	80.13	0.77	0.73	0.67	0.99	0.99	
<i>Invariant and Parameter Prediction Methods</i>								
VICReg	84.28	74.71	0.45	0.39	0.22	0.10	0.50	
SimCLR	86.73	80.08	0.52	0.44	0.25	0.29	0.50	
SimCLR + AugSelf	<b>87.44</b>	<b>80.86</b>	<b>0.75</b>	<b>0.69</b>	<b>0.50</b>	0.28	0.51	
<i>Equivariant Methods</i>								
SEN	86.99	80.20	0.51	0.45	0.26	0.29	0.47	
EquiMod	<b>87.39</b>	<b>80.76</b>	0.50	0.43	0.24	0.29	0.38	
SIE	82.94	75.56	0.73	0.45	0.20	0.07	0.46	
CapsIE	79.14	75.20	0.74	0.60	0.46	0.01	0.03	
EquiCaps	83.24	76.91	<b>0.78</b>	<b>0.73</b>	<b>0.60</b>	0.09	0.05	

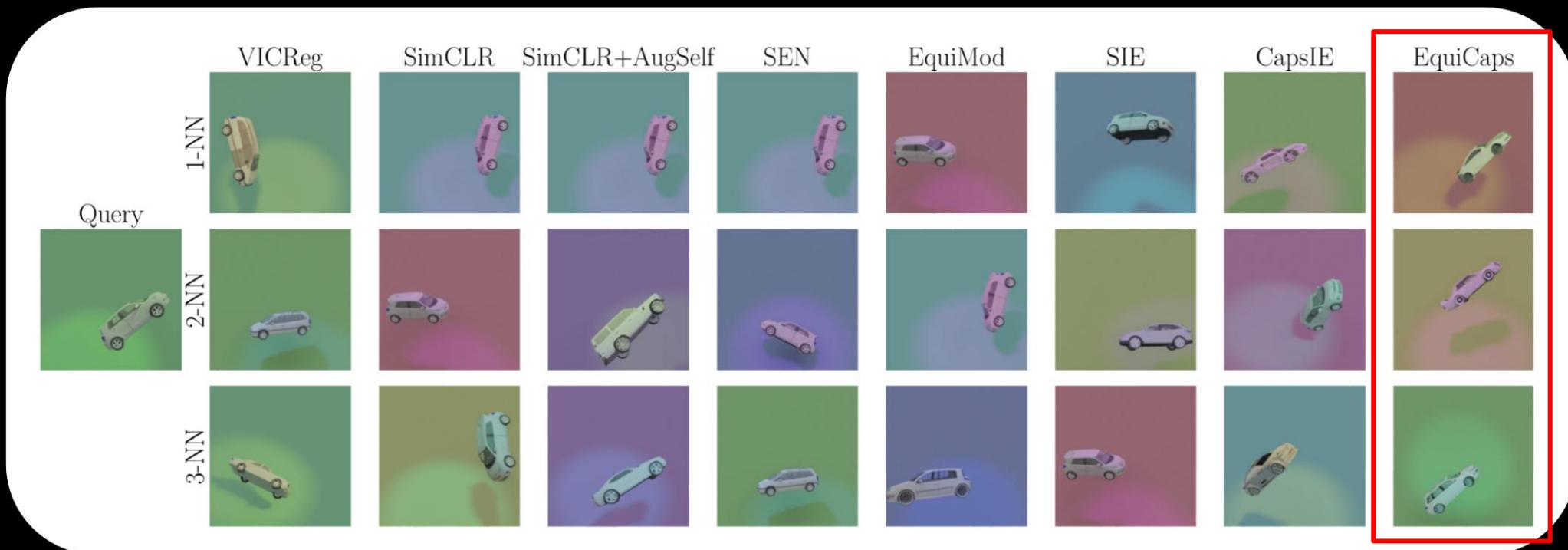
# QUANTITATIVE RESULTS

Pre-train for rotation & translation equivariance

Method	Classification (Top-1)	Rotation ( $R^2$ )	Translation ( $R^2$ )	Colour ( $R^2$ )
SimCLR + AugSelf	81.04 $\uparrow$ 0.18	0.69 = 0.00	0.64 $\uparrow$ 0.14	0.51 = 0.00
SEN	80.23 $\uparrow$ 0.03	0.46 $\uparrow$ 0.01	0.28 $\uparrow$ 0.02	0.50 $\uparrow$ 0.03
EquiMod	<b>80.89</b> $\uparrow$ 0.13	0.46 $\uparrow$ 0.03	0.37 $\uparrow$ 0.13	0.37 $\downarrow$ 0.01
SIE	75.91 $\uparrow$ 0.35	0.48 $\uparrow$ 0.03	0.22 $\uparrow$ 0.02	0.36 $\downarrow$ 0.10
CapsIE	76.31 $\uparrow$ 1.11	0.62 $\uparrow$ 0.02	0.53 $\uparrow$ 0.07	0.03 = 0.00
EquiCaps	77.88 $\uparrow$ 0.97	<b>0.71</b> $\downarrow$ 0.02	<b>0.61</b> $\uparrow$ 0.01	0.02 $\downarrow$ 0.03

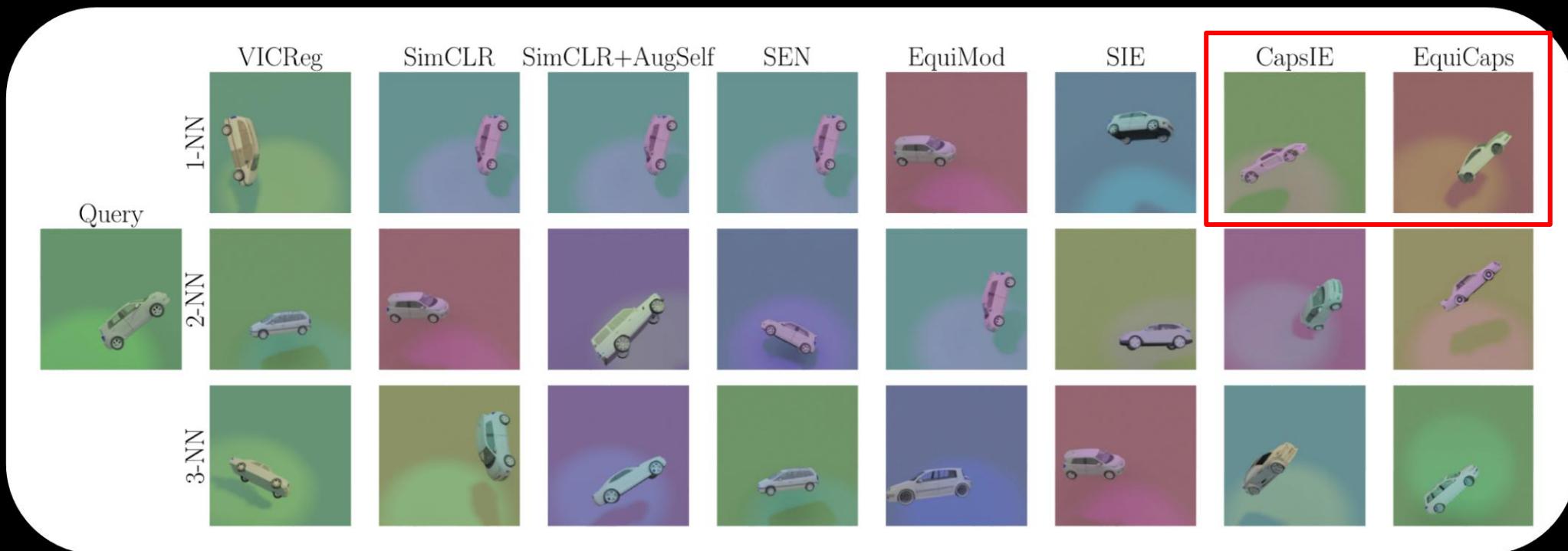
# QUALITATIVE RESULTS

$k$ -NN representation retrieval



# QUALITATIVE RESULTS

$k$ -NN representation retrieval



# MAIN TAKEAWAYS

- **EquiCaps** (predictor-free equivariance)
  - Capsule-based projector
  - Controllable and interpretable latent space
- 3DIEBench-T (**SE(3)** benchmark)
- Extensive experiments
  - **SOTA** on rotation and translation prediction among the equivariant baselines
  - Capsule architectures show **improved generalisation** under combined SE(3) transformations and in transfer learning (including object detection)

# THANK YOU



ArXiv



Code



Dataset