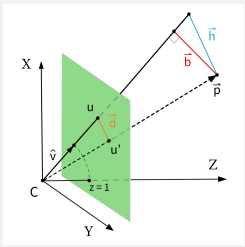
DRaM-LHM: A Quaternion Framework for Iterative Camera Pose Estimation

Chen Lin, Weizhi Du, Zhixiang Min, Baochen She, Enrique Dunn, Sonya M Hanson

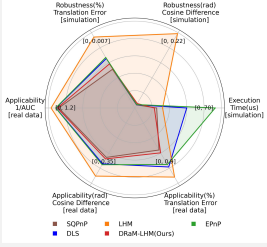
ICCV OCT 19-23, 2025 HONOLULU HAWAII

The PnP Problem

The Perspective-n-Point (PnP) problem estimates a camera's absolute pose from known 3D points and their 2D image projections, given calibrated intrinsics, by minimizing a projective geometry–based objective.



⋆standard objective function



⋆key results

Our Approach

Initialize with solving a closed-form **orthographic projection**, then iteratively refine by minimizing the collinearity error **b**.

Our Contribution

Extend an orthographic formulation to the standard PnP setting, enabling full 6-DoF pose estimation.

By controlling the number of iterations, our framework can either match/surpass SOTA accuracy or deliver SOTA runtime efficiency.

https://github.com/flatironinstitute/capture_lock.git

Method Overview

3D-2D data → Ortho Init (DRaM closed form) → Iterative → Refinement(LHM) → Final Pose

Adjugate Quaternion

⋆Adjugate variables parametrize the rotation matrix

$$R(q_{ij})=\begin{bmatrix} q_{00}+q_{11}-q_{22}-q_{33} & 2q_{12}-2q_{03} & 2q_{13}+2q_{02} \\ 2q_{12}+2q_{03} & q_{00}-q_{11}+q_{22}-q_{33} & 2q_{23}-2q_{01} \\ 2q_{13}-2q_{02} & 2q_{23}+2q_{01} & q_{00}-q_{11}-q_{22}+q_{33} \end{bmatrix}$$

⋆Adjugate variables are quadratic terms of unit quaternion

$$q_{ij} = q_i q_j$$

⋆Adjugate quaternion defines a quadratic mapping $R^{10} \rightarrow S^3$ manifold, subject to **7 constraints** to remain on the unit 3-sphere.

$$\sum_{i=0}^{3} q_{ii}=1$$
$$q_{00}q_{ii}=q_{0i}^2, i=1,2,3$$
$$q_{ii}q_{33}=q_{i3}^2, i=1,2,3$$

Determinant Ratio Matrix (DRaM)

orthographic reprojection error:

$$e_{ORTHO}=\sum_{k=1}^{K}\left\|\boldsymbol{P}(q_{ij})\,\boldsymbol{p}_k+\boldsymbol{t}_{1:2}-\boldsymbol{u}_k\right\|^2$$

polynomial w/ 2D & 3D data terms

$$S_{ORTHO}(q_{ij};x,y,z;u,v)$$

optimal conditions

$$\sum_{i=0}^{3} q_{ii}=1;\frac{\partial S_{ORTHO}}{dq_{ij}}=0$$

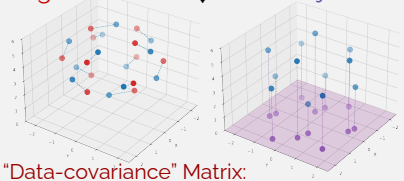
closed-form solution w/ DRaM

$$R(r_1,r_2,r_1\times r_2)=\begin{bmatrix} \frac{d_{xzy}}{\sigma_0} & \frac{d_{zzy}}{\sigma_0} & \frac{d_{yzy}}{\sigma_0} \\ \frac{d_{yzy}}{\sigma_0} & \frac{d_{zzy}}{\sigma_0} & \frac{d_{xzy}}{\sigma_0} \\ \frac{d_{xyy}}{\sigma_0} & \frac{d_{yyy}}{\sigma_0} & \frac{d_{zyy}}{\sigma_0} \end{bmatrix}=\begin{bmatrix} r_1 \\ r_2 \\ r_1\times r_2 \end{bmatrix}$$

Determinant Ratio Matrix (DRaM)

$$\sigma_0=d_{xyz} \rightarrow \det\begin{bmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{bmatrix}$$

Original → Rotated ↓Ortho-Projected

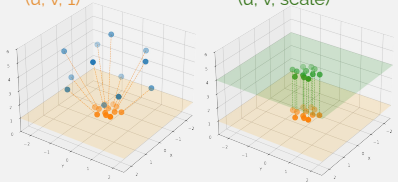


"Data-covariance" Matrix:

$$\begin{bmatrix} xx & xy & xz & ux & vx \\ xy & yy & yz & uy & vy \\ xz & yz & zz & uz & vz \\ ux & uy & uz & uu & uv \\ vx & vy & vz & vu & vv \end{bmatrix} = \begin{bmatrix} \Lambda x & \Lambda\bar{x} \\ \Lambda y & \Lambda\bar{y} \\ \Lambda z & \Lambda\bar{z} \\ \Lambda u & \Lambda\bar{u} \\ \Lambda v & \Lambda\bar{v} \end{bmatrix}$$

← determinant of submatrix

↓ Perspective-Projected (u, v, 1) ↑Weak Persp Approx (u, v, scale)



Iterative Refinement and LHM

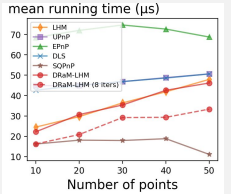
Collinearity error **b**

$$e_{LHM}=\sum_{k=1}^{K}\left\|(R\,\boldsymbol{p}_k+\boldsymbol{t})-\bar{v}_k\left(\bar{v}_k^{\top}(R\,\boldsymbol{p}_k+\boldsymbol{t})\right)\right\|^2$$

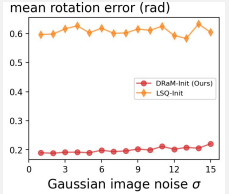
Optimal translation when fixing **R**

$$t(R)=\frac{1}{n}\left(I-\frac{1}{n}\bar{v}_k\right)\cdot\sum_{k=1}^{K}(\bar{v}_k-I)\cdot R\cdot\boldsymbol{p}_k$$

Results



⋆DRaM-LHM achieves SOTA runtime when tuning number of point pairs or iterations



⋆DRaM provides significantly more accurate initialization than weak-perspective

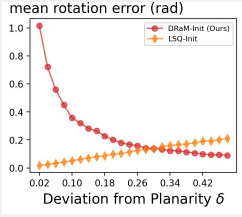
mean rotation error (rad): $2\arccos(q_{est}q_{g.t.})$

Seq	fr1/teddy	fr2/360	MH01
EPnP	0.0035	0.0026	0.0218
SQPnP	0.003	0.0023	0.0206
DRaM	0.0031	0.0024	0.0194

AUC (Areas Under Curver)

Seq	fr1/teddy	fr2/360	MH01
EPnP	0.9156	0.8728	0.5286
SQPnP	0.9558	0.9069	0.5755
DRaM	0.9353	0.8826	0.5708

Limitations and Planar Case



⋆Vanishing determinant σ_0 can cause unstable estimates.

⋆Works well on well-distributed data; falls back to weak perspective if quasi-planar.